Demand Response and Dynamic Pricing in the Smart Grid: Efficiency, Fairness and Robustness

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Introduction: Demand Response



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Demand Response and Dynamic Pricing

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(increasing and convex function of total load).



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- Aggregator broadcast costs and aggregated load $(\ell^t)_t$,
- Consumers eventually reach an equilibrium.

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where b_n is the dynamic price for user n (bill), taken as:

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 \rightarrow N-person minimization game $\mathcal{G} := (\mathcal{N}, \mathcal{L}, (b_n)_n)$

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NASH EQUILIBRIUM (NE) $(\ell_n)_n$ is a NE *IFF* for all *n*:

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$$\begin{array}{l} \text{Definition (Price of Anarchy)}\\\\ \text{PoA}(\mathcal{G}) := \frac{\sup_{\boldsymbol{\ell} \in \mathcal{X}_{\mathcal{G}}^{\text{NE}}} \operatorname{SC}(\boldsymbol{\ell})}{\operatorname{SC}(\boldsymbol{\ell}^{*})} \ , \end{array}$$

where $SC(.) = \sum_{n} b_{n}(.)$ is the social cost.

• with *Daily* billing b_n^{DP} , every user minimizes $\frac{E_n}{E} \sum_{t \in \mathcal{T}} C_t(\ell^t)$ (see Mohsenian-Rad et al., 2010)

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Theorem (J. et al., 2017)

Assume costs are quadratic:

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Theorem (J. et al., 2017)

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Then the PoA is upper bounded:

$$\mathsf{PoA} \leq 1 + rac{3}{4} \sup_{h \in \mathcal{H}} rac{1}{1 + a_1^h/(a_2^h \overline{\ell}^h)}.$$

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Fairness: Prices and bills sould be fair and attractive to users,

Fair's fair

Externality brought by n:

$$V_n := \mathcal{C}^*_{\mathcal{N}} - \mathcal{C}^*_{\mathcal{N} \setminus \{n\}}$$

Which would be equilibrium payments of the billing system:

$$b_n^{\scriptscriptstyle ext{VCG}}(\mathbf{x}_n,\mathbf{x}_{-n}) := \sum_{h \in \mathcal{H}} C_h\left(\sum_{m \in \mathcal{N}} \ell_m^h\right) - \mathcal{C}^*_{\mathcal{N} \setminus \{n\}}$$

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Definition (Baharlouei and Hashemi, 2014)

The fairness index of a billing mechanism $(b_n)_n$ is its maximal normalized distance to $(V_n)_n$ at a Nash Equilibrium:

$$F := \sup_{\mathbf{x} \in \mathcal{X}_{\mathcal{G}}^{\text{NE}}} \left| \sum_{n \in \mathcal{N}} \left| \frac{V_n}{\sum_{m \in \mathcal{N}} V_m} - \frac{b_n(\mathbf{x})}{\sum_{m \in \mathcal{N}} b_m(\mathbf{x})} \right| \right|.$$
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 \rightarrow Relation to Shapley Value

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Cost of constraints

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Efficiency Versus Fairness

Simulation: 30 days, 30 users EV owners (Data Pecan Street Inc.)



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Assume user's objective is modified as:

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What is the impact on the equilibrium profile and global system costs ?

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Proposition (Jacquot et al., 2017)

Assume $\forall n \in \mathcal{N}$, $\frac{\hat{\ell}_n^p}{E_n} + \frac{1}{2} \geq \frac{\hat{\ell}^p}{E}$, then, for $\alpha \in (0, 1]$, the NE of $\mathcal{G}_{\alpha}^{DP}$ gives:

$$\forall h \in \{P, O\}, \ \ell^h = E/2 + \alpha \times (\hat{\ell}^{\bar{h}} - \hat{\ell}^{\bar{h}})/2 .$$
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Proposition (Jacquot et al., 2017)

Assume $\forall n \in \mathcal{N}$, $\hat{\ell}_n^P \geq \frac{(\hat{\ell}^P - \hat{\ell}^O) - E_n}{2(N-1)}$, then $\forall \alpha \in [0, 1]$, the NE of $\mathcal{G}_{\alpha}^{HP}$ gives:

$$\forall h \in \{P, O\}, \ \ell^h = E/2 + \phi(\alpha) \times (\hat{\ell}^h - \hat{\ell}^{\bar{h}})/2 \ . \tag{4}$$

where $\phi(\alpha) \stackrel{\text{def}}{=} \frac{2\alpha}{(1+\alpha)+(1-\alpha)N} \in [0,1].$

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THANK YOU!

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