Efficiency of a Demand Response Game-Theoretic Model for the Smart Grid

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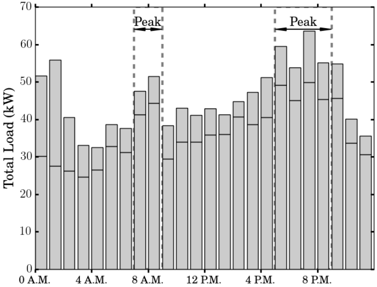
Paris Game Theory Ph.D. Seminar

Introduction: Demand Response



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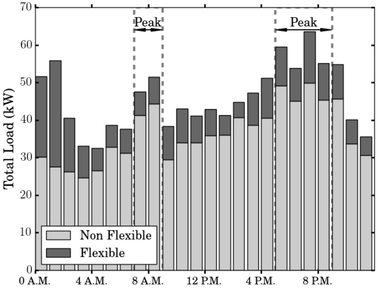

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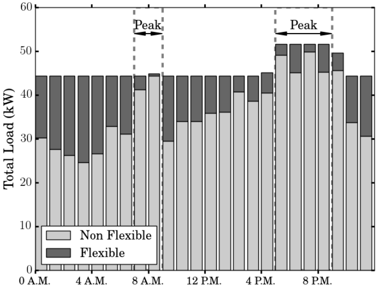

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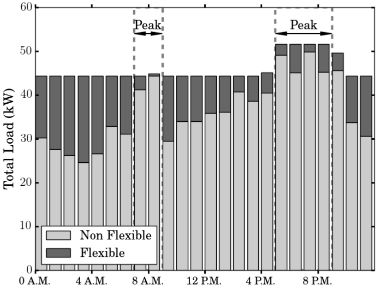

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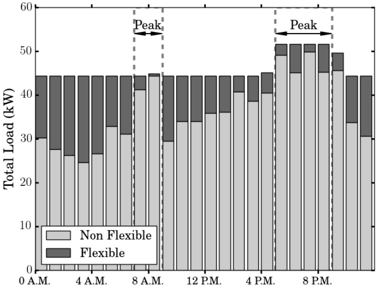

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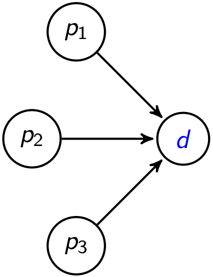
What efficiency can be achieved here?

Model Autonomous Network



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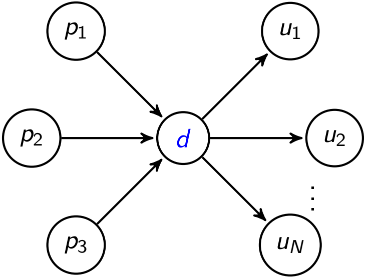

- One distributor/aggregator d sources from energy producers and market.
- d provides energy to a set $\mathcal{N}$ of N residential consumers

Modeling Users

Assume that the provider's costs are increasing and convex functions of total load ℓ^h at each time period h, e.g.:

$$C_h(\ell^h) = a_{h2}(\ell^h)^2 + a_{h1}\ell^h + a_{h0}, \tag{1}$$

Each user n has a set of **flexible** items $\mathcal{A}_n$ (EV, heater,..), each $a \in \mathcal{A}_n$:

- requires a fix daily energy E_{na},
- can be used during a subset of time periods $\mathcal{A}_n = \{\alpha_{na}, \ldots, \beta_{na}\}$,
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Each user pays a daily bill b_n for its consumption $\boldsymbol{\ell}_\mathbf{n} = (\ell_n^h)_h$. What is the *good* signal b_n to send to users ?

Daily proportional billing

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- the NE achieves optimality (PoA=0): minimizing social cost $SC = \sum_n b_n = \sum_h C_h(\ell^h)$
- flexible users may not have strong incentives and pay for others that make hours really expensive:
  If n add an extra load E_n on a very expensive hour h, he adds a cost $V_n = C_h(\ell^h + E_n) - C_h(\ell^h)$ and pay for it $\dfrac{E_n}{\sum_m E_m}$, the remaining being paid by the others.

Hourly proportional billing

- $b_n = \kappa \sum_h \left(\frac{\ell_n^h}{\sum_m \ell_m^h} C_h(\ell^h) \right) = \kappa \sum_h \left(\ell_n^h c_h(l^h) \right)$
  with $c_h(\ell^h) = C^h(\ell^h)/\ell^h$ the per-unit price of energy.
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Theorem

If $c_h'(\ell^h) \geq 0$ *and* $\forall h, \frac{(\ell^h)^2}{\sum_n (\ell_n^h)^2} > \left(\frac{\ell^h c_h''(\ell^h)}{2 c_h'(\ell^h)} \right)^2$
then there is a **unique** *load per user* $(\boldsymbol{\ell}_n)_{n \in \mathcal{N}}$ *that provides a NE.*

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Proof on board!

Price Of Anarchy

$$\mathrm{PoA}(\mathcal{G}) := \frac{\sup_{\mathbf{x} \in \mathcal{X}_{\mathcal{G}}^{\mathrm{NE}}} \mathrm{SC}(\mathbf{x})}{\mathrm{SC}^*} \ . \tag{2}$$

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Definition (Local Smoothness, Roughgarden and Schoppmann)

A cost minimization game $\mathcal{G} = (\mathcal{N}, \mathcal{X}, (b_n)_n)$ is locally (λ, μ)-smooth with respect to y iff for all feasible outcome x:

$$\sum_{n\in\mathcal{N}} b_n(x) + \nabla_n b_n(x)^T(y_n - x_n) \leq \lambda \mathrm{SC}(y) + \mu \mathrm{SC}(x) \ .$$

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Theorem (Roughgarden and Schoppmann)

If a game is $(\lambda, \mu)-$smooth with any optimal outcome y then the PoA of any correlated equilibrium is at most $\frac{\lambda}{1-\mu}$.

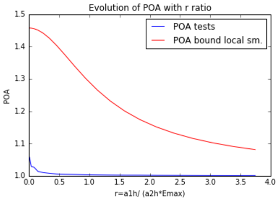
Using local smoothness and multiple lines of calculus, we get:

Theorem

With quad. cost functions $C_h(\ell^h) = a_{h2}(\ell^h)^2 + a_{h1}\ell^h$ *and* $r = \sup_h \frac{a_1^h}{a_2^h \cdot \bar{l}^h}$,

$$PoA \leq \frac{1}{2}\left(1 + \sqrt{1 + \frac{1}{(1+r)^2} + \frac{1}{2(1+r)}}\right)$$

and the upper bound is $\underset{r\to\infty}{\sim} 1 + \frac{1}{4r}$.



Fairness

Why Fairness ?

Let $\mathtt{SC}^*_{\mathcal{M}}$ be the minimal system costs achievable with the subset of users $\mathcal{M}$.
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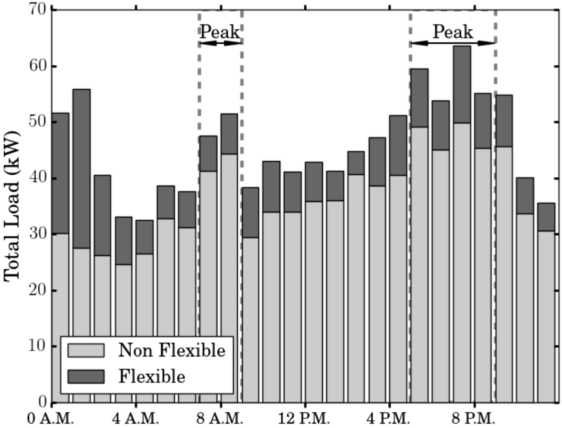
We define the **fairness** of billing $(b_n)_n$ as:

$$F = \sum_{n \in \mathcal{N}} \left| \frac{V_n}{\sum_m V_m} - \frac{b_n}{\sum_m b_m} \right| \tag{3}$$

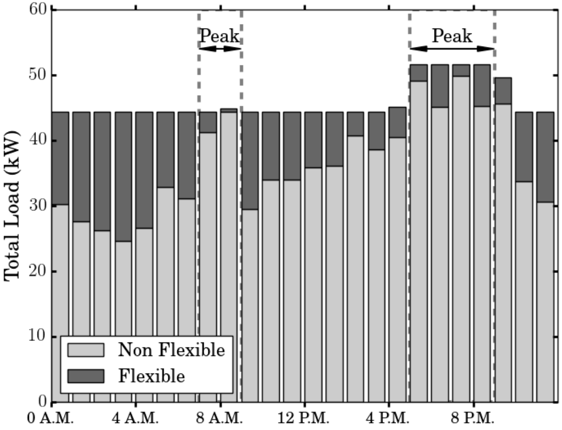
Load Profile

Data/Default Profile



Load Profile

DLP equilibrium Profile (SC optimum)



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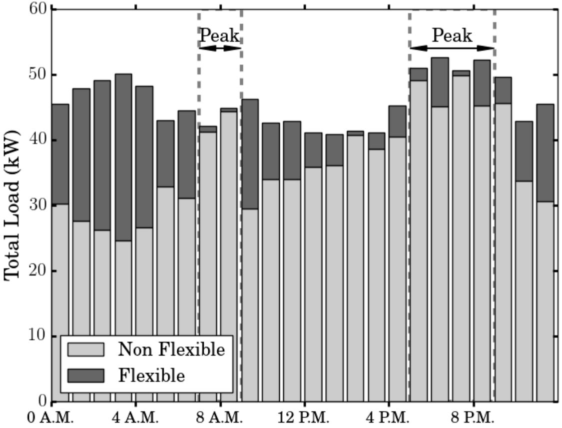
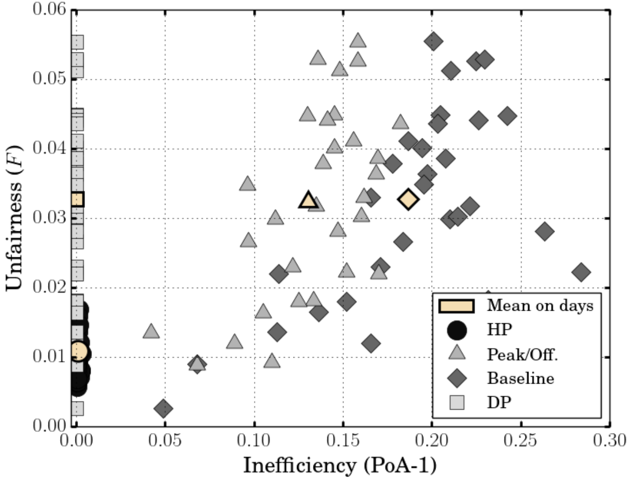
HLP equilibrium Profile



Efficiency versus Fairness



Consumers utility

→ user n has a preferred profile $\hat{\mathbf{x}}_{na} = (\hat{x}_{na}^h)_h$ for each of his flexible appliance $a \in \mathcal{A}_n$.
→ Disutility:

$$u_n(\mathbf{x}^n) \stackrel{\text{def}}{=} -\sum_{a \in \mathcal{A}_n} \omega_{na}^h \sum_h \left\| \mathbf{x}_n^h - \hat{\mathbf{x}}_n^h \right\|^2 \qquad (4)$$

→ user n now aims at minimizing:

$$\pi_n(\mathbf{x}_n) \stackrel{\text{def}}{=} b_n(\mathbf{x}) - u_n(\mathbf{x}_n) \qquad (5)$$

while satisfying all previous constraints $\mathbf{x} \in \mathcal{X}_n$. The social cost of the system is therefore modified as:

$$\mathrm{SC}(\mathbf{x}) = \sum_n \pi_n(\mathbf{x}) = \sum_n b_n(\mathbf{x}) - u_n(\mathbf{x}_n) \qquad (6)$$

Convergence of the BR process

To implement this process, we should ask for a fast convergence of the algorithm:

- With quadratic system costs functions $C_h = a_1^h \ell^h + a_2^h (\ell^h)^2$, we have a potential game
  $\rightarrow$ the BRD will converge to the NE in $\mathcal{O}(\frac{1}{K})$
- Can we have a convergence result in the general case?

Theorem (Asymptotic Convergence of the continuous BRD)

We consider the dynamics: $\dot{x}_n(t) = BR_n(x_{-n}(t)) - x_n(t)$. *If the game is dissipative, then* $x(t)$ *is asymptotically convergent, and* $H(x) = \max_{y \in X} \langle y - x, \Phi(x) \rangle$ *is a Lyapunov function.*

Can we have a convergence result of the discrete dynamic?

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THANK YOU