A Game-theoretic Model for Demand Response: Analysis, Implementation and Challenges

P. Jacquot [1,2] O.Beaude [1] S. Gaubert [2] N.Oudjane [1]

[1]EDF Lab Saclay

[2]Inria and CMAP, École polytechnique

January 6, 2018

Joint seminar UK Center-EDF Lab

Storage and load flexibility control

EDF Lab Saclay

Demand Response: exploiting consumer flexibilities

Set of techniques to exploit consumers **flexibilities**.

Different goals:

Demand Response: exploiting consumer flexibilities

Set of techniques to exploit consumers **flexibilities**.

Different goals:

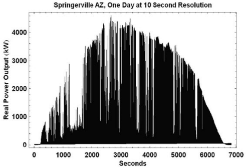
- reducing production peaks and peak power plants,

Demand Response: exploiting consumer flexibilities

Set of techniques to exploit consumers **flexibilities**.

Different goals:

- reducing production peaks and peak power plants,
- increasing distributed renewable energy insertion,



Demand Response: exploiting consumer flexibilities

Set of techniques to exploit consumers **flexibilities**.

Different goals:

- reducing production peaks and peak power plants,
- increasing distributed renewable energy insertion,
- avoiding congestion on the power grid.

Why considering a Game Theory approach?

- **individual agents** (residential/industrial consumers) or aggregators make consumption **decisions** based on price incentives and personal utilities,

Why considering a Game Theory approach?

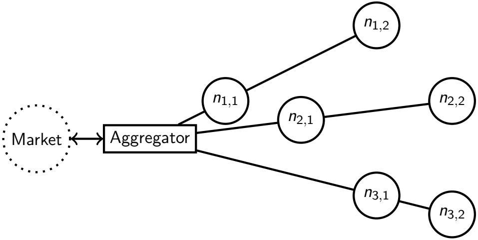
- **individual agents** (residential/industrial consumers) or aggregators make consumption **decisions** based on price incentives and personal utilities,
- individual decisions have an **impact** on the distribution level, and **affect** the behavior of others (congestion on the network, market prices) ,

Why considering a Game Theory approach?

- **individual agents** (residential/industrial consumers) or aggregators make consumption **decisions** based on price incentives and personal utilities,
- individual decisions have an **impact** on the distribution level, and **affect** the behavior of others (congestion on the network, market prices) ,
- adopting a **decentralized** point of view: distributed optimization for tractability, minimizing the unveiled information of consumers and respect **privacy** constraints.

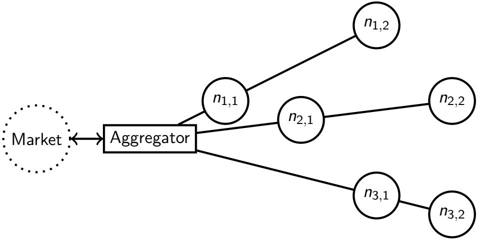
The role of (residential) aggregators

"Optimize" the flexibilities of a pool of consumers



The role of (residential) aggregators

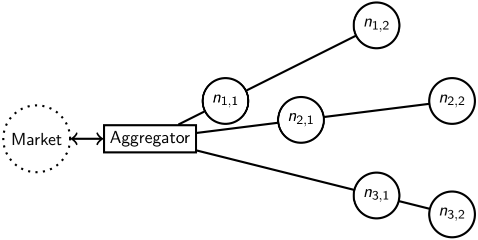
"Optimize" the flexibilities of a pool of consumers



- to directly give electricity to those consumers at lower rates by interacting with markets,

The role of (residential) aggregators

"Optimize" the flexibilities of a pool of consumers



- to directly give electricity to those consumers at lower rates by interacting with markets,
- to sell flexibility and curtailment to an other market actor or a grid operator.

The model: electricity consumption game

- "optimization" horizon as a finite time set $\mathcal{T} = \{1, \ldots, T\}$.

The model: electricity consumption game

- “optimization” horizon as a finite time set $\mathcal{T} = \{1, \ldots, T\}$.
- district of N flexible consumers $\mathcal{N} = \{1, \ldots, N\}$

The model: electricity consumption game

- "optimization" horizon as a finite time set $\mathcal{T} = \{1, \dots, T\}$.
- district of N flexible consumers $\mathcal{N} = \{1, \dots, N\}$
  - with nonflexible consumption (fix demand) profile (lighting, cooking, TV set...) denoted $(D_{n,t})_{t \in \mathcal{T}}$

The model: electricity consumption game

- "optimization" horizon as a finite time set $\mathcal{T} = \{1, \dots, T\}$.
- district of N flexible consumers $\mathcal{N} = \{1, \dots, N\}$
  - with nonflexible consumption (fix demand) profile (lighting, cooking, TV set...) denoted $(D_{n,t})_{t\in\mathcal{T}}$
  - and flexible consumption (Electric vehicle charging, water heater, washing machine,...): variable profile, controlled by an ECS, denoted $(\ell_{n,t})_{t\in\mathcal{T}}$

The model: electricity consumption game

- "optimization" horizon as a finite time set $\mathcal{T} = \{1, \dots, T\}$.
- district of N flexible consumers $\mathcal{N} = \{1, \dots, N\}$
  - with nonflexible consumption (fix demand) profile (lighting, cooking, TV set...) denoted $(D_{n,t})_{t \in \mathcal{T}}$
  - and flexible consumption (Electric vehicle charging, water heater, washing machine,...): variable profile, controlled by an ECS, denoted $(\ell_{n,t})_{t \in \mathcal{T}}$
- one aggregator gets the **aggregated** nonflexible demand $D_t = \sum_{n \in \mathcal{N}} D_{n,t}$ and flexible profile $L_t \stackrel{\text{def}}{=} \sum_{n \in \mathcal{N}} \ell_{n,t}$.

Aggregator price functions for flexible load

For each t, the aggregator has a per-unit energy price function $L_t \mapsto c_t(L_t)$
Examples:

- minimize distance to vector of bid quantities $(Q_t)_{t\in\mathcal{T}}$ on DA market
- minimizing production costs with self production.

Aggregator price functions for flexible load

For each t, the aggregator has a per-unit energy price function $L_t \mapsto c_t(L_t)$

Examples:

- minimize distance to vector of bid quantities $(Q_t)_{t\in\mathcal{T}}$ on DA market
- minimizing production costs with self production.

Remark

production costs $C_t(.) \Leftrightarrow$ *prices* $c_t(.)$: $c_t(L_t) \overset{def}{=} \frac{1}{L_t}\Big[C_t(D_t + L_t) - C_t(D_t)\Big]$.

Aggregator price functions for flexible load

For each t, the aggregator has a per-unit energy price function $L_t \mapsto c_t(L_t)$
Examples:

- minimize distance to vector of bid quantities $(Q_t)_{t\in\mathcal{T}}$ on DA market
- minimizing production costs with self production.

Remark

production costs $C_t(.) \Leftrightarrow$ *prices* $c_t(.)$: $c_t(L_t) \stackrel{def}{=} \frac{1}{L_t}\Big[C_t(D_t + L_t) - C_t(D_t)\Big]$.

general assumption: $c_t(.)$ smooth (D2), convex, strictly increasing

Aggregator price functions for flexible load

For each t, the aggregator has a per-unit energy price function $L_t \mapsto c_t(L_t)$
Examples:

- minimize distance to vector of bid quantities $(Q_t)_{t\in\mathcal{T}}$ on DA market
- minimizing production costs with self production.

Remark

production costs $C_t(.) \Leftrightarrow$ *prices* $c_t(.)$: $c_t(L_t) \overset{def}{=} \frac{1}{L_t}\Big[C_t(D_t + L_t) - C_t(D_t)\Big]$.

general assumption: $c_t(.)$ smooth (D2), convex, strictly increasing

Remark: for instance affine prices $\forall t \in \mathcal{T},\ c_t(\ell) = \alpha_t + \beta_t \ell$ with $\alpha_t, \beta_t \in (\mathbb{R}_+^*)^2$.

A Flexible Consumer's constraints and model

For each consumer $n \in \mathcal{N}$:

- n has a flexible energy demand E_n to be satisfied over $\mathcal{T}$ (e.g. EV battery),

$$\sum_{t \in \mathcal{T}} \ell_{n,t} = E_n \ , \tag{1c}$$

A Flexible Consumer's constraints and model

For each consumer $n \in \mathcal{N}$:

- n has a flexible energy demand E_n to be satisfied over $\mathcal{T}$ (e.g. EV battery),
- for each time period t, the flexible load is bounded between $\underline{\ell}_{n,t}$ and $\overline{\ell}_{n,t}$

$$\sum_{t \in \mathcal{T}} \ell_{n,t} = E_n \ , \tag{1b}$$

$$\underline{\ell}_{n,t} \leq \ell_{n,t} \leq \overline{\ell}_{n,t}, \forall t \in \mathcal{T} \ . \tag{1c}$$

A Flexible Consumer's constraints and model

For each consumer $n \in \mathcal{N}$:

- n has a flexible energy demand E_n to be satisfied over $\mathcal{T}$ (e.g. EV battery),
- for each time period t, the flexible load is bounded between $\underline{\ell}_{n,t}$ and $\overline{\ell}_{n,t}$
  - **rm 1:** agg. constraints of appliances, unavailability at time periods

$$\sum_{t\in\mathcal{T}} \ell_{n,t} = E_n \ , \tag{1b}$$

$$\underline{\ell}_{n,t} \leq \ell_{n,t} \leq \overline{\ell}_{n,t}, \forall t \in \mathcal{T} \ . \tag{1c}$$

A Flexible Consumer's constraints and model

For each consumer $n \in \mathcal{N}$:

- n has a flexible energy demand E_n to be satisfied over $\mathcal{T}$ (e.g. EV battery),
- for each time period t, the flexible load is bounded between $\underline{\ell}_{n,t}$ and $\overline{\ell}_{n,t}$
  - **rm 1:** agg. constraints of appliances, unavailability at time periods
  - **rm 2:** more complex models exist: thermostatic regulation loads, "block" loads, discrete power levels.

$$\sum_{t\in\mathcal{T}} \ell_{n,t} = E_n \ , \tag{1b}$$

$$\underline{\ell}_{n,t} \leq \ell_{n,t} \leq \overline{\ell}_{n,t}, \forall t \in \mathcal{T} \ . \tag{1c}$$

A Flexible Consumer's constraints and model

For each consumer $n \in \mathcal{N}$:

- n has a flexible energy demand E_n to be satisfied over $\mathcal{T}$ (e.g. EV battery),
- for each time period t, the flexible load is bounded between $\underline{\ell}_{n,t}$ and $\overline{\ell}_{n,t}$
  - **rm 1:** agg. constraints of appliances, unavailability at time periods
  - **rm 2:** more complex models exist: thermostatic regulation loads, "block" loads, discrete power levels.
- consumer minimizes the **bill** $b_n(\boldsymbol{\ell}_n, \boldsymbol{\ell}_{-n}) \stackrel{\text{def}}{=} \sum_{t \in \mathcal{T}} \ell_{n,t} c_t(L_t)$

$$\min_{\boldsymbol{\ell}_n \in \mathbb{R}^T} b_n(\boldsymbol{\ell}_n, \boldsymbol{\ell}_{-n}) \tag{1a}$$

$$\text{s.t.} \quad \sum_{t \in \mathcal{T}} \ell_{n,t} = E_n \ , \tag{1b}$$

$$\underline{\ell}_{n,t} \leq \ell_{n,t} \leq \overline{\ell}_{n,t}, \forall t \in \mathcal{T} \ . \tag{1c}$$

A Flexible Consumer's constraints and model

For each consumer $n \in \mathcal{N}$:

- n has a flexible energy demand E_n to be satisfied over $\mathcal{T}$ (e.g. EV battery),
- for each time period t, the flexible load is bounded between $\underline{\ell}_{n,t}$ and $\overline{\ell}_{n,t}$
  - **rm 1:** agg. constraints of appliances, unavailability at time periods
  - **rm 2:** more complex models exist: thermostatic regulation loads, "block" loads, discrete power levels.
- consumer minimizes the **bill** $b_n(\boldsymbol{\ell}_n, \boldsymbol{\ell}_{-n}) \stackrel{\text{def}}{=} \sum_{t \in \mathcal{T}} \ell_{n,t} c_t(L_t)$

$$\min_{\boldsymbol{\ell}_n \in \mathbb{R}^T} b_n(\boldsymbol{\ell}_n, \boldsymbol{\ell}_{-n}) \tag{1a}$$

$$\text{s.t.} \quad \sum_{t \in \mathcal{T}} \ell_{n,t} = E_n \ , \tag{1b}$$

$$\underline{\ell}_{n,t} \leq \ell_{n,t} \leq \overline{\ell}_{n,t}, \forall t \in \mathcal{T} \ . \tag{1c}$$

- **rm 1:** depends on the profile of others through L_t

A Flexible Consumer's constraints and model

For each consumer $n \in \mathcal{N}$:

- n has a flexible energy demand E_n to be satisfied over $\mathcal{T}$ (e.g. EV battery),
- for each time period t, the flexible load is bounded between $\underline{\ell}_{n,t}$ and $\overline{\ell}_{n,t}$
  - **rm 1:** agg. constraints of appliances, unavailability at time periods
  - **rm 2:** more complex models exist: thermostatic regulation loads, "block" loads, discrete power levels.
- consumer minimizes the **bill** $b_n(\boldsymbol{\ell}_n, \boldsymbol{\ell}_{-n}) \stackrel{\text{def}}{=} \sum_{t\in\mathcal{T}} \ell_{n,t} c_t(L_t)$

$$\min_{\boldsymbol{\ell}_n \in \mathbb{R}^T} b_n(\boldsymbol{\ell}_n, \boldsymbol{\ell}_{-n}) \tag{1a}$$

$$\text{s.t.} \quad \sum_{t\in\mathcal{T}} \ell_{n,t} = E_n \ , \tag{1b}$$

$$\underline{\ell}_{n,t} \leq \ell_{n,t} \leq \overline{\ell}_{n,t}, \forall t \in \mathcal{T} \ . \tag{1c}$$

- **rm 1:** depends on the profile of others through L_t
- **rm 2:** simple billing mechanism, aggregator could optimize its pricing.

Nash Equilibrium

- The minimization of each player depends on the others decisions!

Nash Equilibrium

- The minimization of each player depends on the others decisions!
- concept of "optimality" $\rightarrow$ Nash Equilibrium $\boldsymbol{\ell}^{\mathrm{NE}}$

Nash Equilibrium

- The minimization of each player depends on the others decisions!
- concept of "optimality" $\rightarrow$ Nash Equilibrium $\boldsymbol{\ell}^{\mathrm{NE}}$

$$\forall \boldsymbol{\ell}_n \in \mathcal{L}_n, \;\; b_n(\boldsymbol{\ell}_n^{\mathrm{NE}}, \boldsymbol{\ell}_{-n}^{\mathrm{NE}}) \leq b_n(\boldsymbol{\ell}_n, \boldsymbol{\ell}_{-n}^{\mathrm{NE}})$$

Nash Equilibrium

- The minimization of each player depends on the others decisions!
- concept of "optimality" $\rightarrow$ Nash Equilibrium $\boldsymbol{\ell}^{\mathrm{NE}}$

$$\begin{aligned} & \forall \boldsymbol{\ell}_n \in \mathcal{L}_n,\ b_n(\boldsymbol{\ell}_n^{\mathrm{NE}}, \boldsymbol{\ell}_{-n}^{\mathrm{NE}}) \leq b_n(\boldsymbol{\ell}_n, \boldsymbol{\ell}_{-n}^{\mathrm{NE}}) \\ \Leftrightarrow \quad & \boldsymbol{\ell}_n^{\mathrm{NE}} \in \operatorname*{arg\,min}_{\boldsymbol{\ell}_n \in \mathcal{L}_n} b_n(\boldsymbol{\ell}_n, \boldsymbol{\ell}_{-n}^{\mathrm{NE}}) \end{aligned}$$

Nash Equilibrium

- The minimization of each player depends on the others decisions!
- concept of "optimality" $\rightarrow$ Nash Equilibrium $\boldsymbol{\ell}^{\mathrm{NE}}$

$$\begin{aligned} & \forall \boldsymbol{\ell}_n \in \mathcal{L}_n,\ b_n(\boldsymbol{\ell}_n^{\mathrm{NE}}, \boldsymbol{\ell}_{-n}^{\mathrm{NE}}) \leq b_n(\boldsymbol{\ell}_n, \boldsymbol{\ell}_{-n}^{\mathrm{NE}}) \\ \Leftrightarrow \quad & \boldsymbol{\ell}_n^{\mathrm{NE}} \in \operatorname*{arg\,min}_{\boldsymbol{\ell}_n \in \mathcal{L}_n} b_n(\boldsymbol{\ell}_n, \boldsymbol{\ell}_{-n}^{\mathrm{NE}}) \end{aligned}$$

- **Existence**

Nash Equilibrium

- The minimization of each player depends on the others decisions!
- concept of "optimality" $\rightarrow$ Nash Equilibrium $\boldsymbol{\ell}^{\mathrm{NE}}$

$$\begin{aligned} & \forall \boldsymbol{\ell}_n \in \mathcal{L}_n,\; b_n(\boldsymbol{\ell}_n^{\mathrm{NE}}, \boldsymbol{\ell}_{-n}^{\mathrm{NE}}) \leq b_n(\boldsymbol{\ell}_n, \boldsymbol{\ell}_{-n}^{\mathrm{NE}}) \\ \Leftrightarrow \quad & \boldsymbol{\ell}_n^{\mathrm{NE}} \in \operatorname*{arg\,min}_{\boldsymbol{\ell}_n \in \mathcal{L}_n} b_n(\boldsymbol{\ell}_n, \boldsymbol{\ell}_{-n}^{\mathrm{NE}}) \end{aligned}$$

- **Existence** and **Uniqueness** .

Measuring sub-optimality: the price of Anarchy

Definition: The social cost is $\mathrm{SC}(\boldsymbol{\ell}) \stackrel{\text{def}}{=} \sum_n b_n(\boldsymbol{\ell})$

Measuring sub-optimality: the price of Anarchy

Definition: The social cost is $\mathrm{SC}(\boldsymbol{\ell}) \stackrel{\text{def}}{=} \sum_n b_n(\boldsymbol{\ell}) = \sum_t L_t c_t(L_t)$.

Measuring sub-optimality: the price of Anarchy

Definition: The social cost is $\mathtt{SC}(\boldsymbol{\ell}) \stackrel{\text{def}}{=} \sum_n b_n(\boldsymbol{\ell}) = \sum_t L_t c_t(L_t)$.

A NE does not necessarily minimize the social cost!

Measuring sub-optimality: the price of Anarchy

Definition: The social cost is $\mathrm{SC}(\boldsymbol{\ell}) \stackrel{\text{def}}{=} \sum_n b_n(\boldsymbol{\ell}) = \sum_t L_t c_t(L_t)$.

A NE does not necessarily minimize the social cost!

If the social cost is high at the equilibrium, our system is inefficient!

Measuring sub-optimality: the price of Anarchy

Definition: The social cost is $\mathrm{SC}(\ell) \stackrel{\text{def}}{=} \sum_n b_n(\ell) = \sum_t L_t c_t(L_t)$.

A NE does not necessarily minimize the social cost!
If the social cost is high at the equilibrium, our system is inefficient!

- Price of Anarchy:

$$\mathrm{PoA}(\mathcal{G}) = \frac{\sup_{\ell \in \mathcal{L}_{\mathrm{NE}}} \mathrm{SC}(\ell)}{\inf_{\ell \in \mathcal{L}} \mathrm{SC}(\ell)} .$$

Measuring sub-optimality: the price of Anarchy

Definition: The social cost is $\mathrm{SC}(\boldsymbol{\ell}) \stackrel{\text{def}}{=} \sum_n b_n(\boldsymbol{\ell}) = \sum_t L_t c_t(L_t)$.

A NE does not necessarily minimize the social cost!
If the social cost is high at the equilibrium, our system is inefficient!

- Price of Anarchy:

$$\mathrm{PoA}(\mathcal{G}) = \frac{\sup_{\boldsymbol{\ell} \in \mathcal{L}_{\mathrm{NE}}} \mathrm{SC}(\boldsymbol{\ell})}{\inf_{\boldsymbol{\ell} \in \mathcal{L}} \mathrm{SC}(\boldsymbol{\ell})} \ .$$

- Bad news: bounding the PoA in general is hard . . .

Measuring sub-optimality: the price of Anarchy

Definition: The social cost is $\mathrm{SC}(\boldsymbol{\ell}) \stackrel{\text{def}}{=} \sum_n b_n(\boldsymbol{\ell}) = \sum_t L_t c_t(L_t)$.

A NE does not necessarily minimize the social cost!
If the social cost is high at the equilibrium, our system is inefficient!

- Price of Anarchy:

$$\mathrm{PoA}(\mathcal{G}) = \frac{\sup_{\boldsymbol{\ell} \in \mathcal{L}_{\mathrm{NE}}} \mathrm{SC}(\boldsymbol{\ell})}{\inf_{\boldsymbol{\ell} \in \mathcal{L}} \mathrm{SC}(\boldsymbol{\ell})} .$$

- Bad news: bounding the PoA in general is hard ...
- ...and it can be arbitrarily high asymptotically, simply with (pos. coeffs) polynomial price functions!

Measuring sub-optimality: the price of Anarchy

Definition: The social cost is $\mathrm{SC}(\boldsymbol{\ell}) \stackrel{\text{def}}{=} \sum_n b_n(\boldsymbol{\ell}) = \sum_t L_t c_t(L_t)$.

A NE does not necessarily minimize the social cost!
If the social cost is high at the equilibrium, our system is inefficient!

- Price of Anarchy:

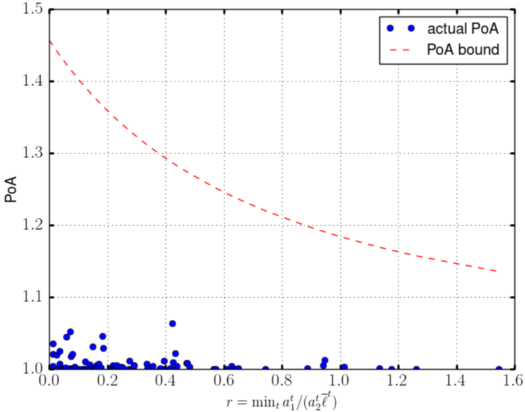
$$\mathrm{PoA}(\mathcal{G}) = \frac{\sup_{\boldsymbol{\ell} \in \mathcal{L}_{\mathrm{NE}}} \mathrm{SC}(\boldsymbol{\ell})}{\inf_{\boldsymbol{\ell} \in \mathcal{L}} \mathrm{SC}(\boldsymbol{\ell})} \ .$$

- Bad news: bounding the PoA in general is hard ...
- ...and it can be arbitrarily high asymptotically, simply with (pos. coeffs) polynomial price functions!
- Good news: the PoA is low with the proposed mechanism!

A bound on the PoA in the affine case

With affine prices for all time periods $c_t(Lt) = \alpha_t + \beta_t L_t$, we have the bound:

$$\mathrm{PoA}(\mathcal{G}) \leq 1 + \frac{3}{4} \sup_{t\in\mathcal{T}} \left(1 + \frac{\alpha_t}{\beta_t \overline{L}_t}\right)^{-1} .$$



Computation of the equilibrium profile

To implement efficiently the Demand Response program, we need that consumers reach the (Nash) equilibrium profiles:

Computation of the equilibrium profile

To implement efficiently the Demand Response program, we need that consumers reach the (Nash) equilibrium profiles:

- autonomous computation: consumers are not spending hours to gain a few euros (autonomous ECS device) . . .

Computation of the equilibrium profile

To implement efficiently the Demand Response program, we need that consumers reach the (Nash) equilibrium profiles:

- autonomous computation: consumers are not spending hours to gain a few euros (autonomous ECS device) . . .
- fast algorithm: scalable for small time steps and large number of consumers, equilibrium can be recomputed several times . . .

Computation of the equilibrium profile

To implement efficiently the Demand Response program, we need that consumers reach the (Nash) equilibrium profiles:

- autonomous computation: consumers are not spending hours to gain a few euros (autonomous ECS device) ...
- fast algorithm: scalable for small time steps and large number of consumers, equilibrium can be recomputed several times ...
- distributed and decentralized: a minimum of private information is exchanged with a coordinator (agg.), consumers keep their private constraints

Two decentralized algorithms

Best Response (BR) Algorithm

Two decentralized algorithms

Best Response (BR) Algorithm

Require: $\ell^{(0)}$, $k_{\max}$, $\varepsilon_{\text{stop}}$,

Two decentralized algorithms

Best Response (BR) Algorithm

Require: $\ell^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},$
$k \leftarrow 0,\ \ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

Two decentralized algorithms

Best Response (BR) Algorithm

Require: $\ell^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},$
$k \leftarrow 0,\ \ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$
while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}$ & $k \leq k_{\max}$ **do**

end while

Two decentralized algorithms

Best Response (BR) Algorithm

Require: $\ell^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},$

$k \leftarrow 0,\ \ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}\ \ \&\ \ k \leq k_{\max}$ **do**

for $n = 1$ to N **do**

end for

end while

Two decentralized algorithms

Best Response (BR) Algorithm

Require: $\boldsymbol{\ell}^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},$

$k \leftarrow 0,\ \ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}$ & $k \leq k_{\max}$ **do**

for $n = 1$ to N **do**

$\boldsymbol{s}_n^{(k)} = \sum_{m<n} \boldsymbol{\ell}_m^{(k+1)} + \sum_{m>n} \boldsymbol{\ell}_m^{(k)}$

$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \mathrm{BR}_n(\boldsymbol{s}_n^{(k)}) = \underset{\boldsymbol{\ell}_n \in \mathcal{L}_n}{\arg\min} \sum_t \ell_{n,t} c_t(s_{n,t} + \ell_{n,t})$

end for

end while

Two decentralized algorithms

Best Response (BR) Algorithm

Require: $\boldsymbol{\ell}^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},$

$k \leftarrow 0,\ \ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}$ & $k \leq k_{\max}$ **do**

for $n = 1$ to N **do**

$\boldsymbol{s}_n^{(k)} = \sum_{m<n} \boldsymbol{\ell}_m^{(k+1)} + \sum_{m>n} \boldsymbol{\ell}_m^{(k)}$

$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \mathrm{BR}_n(\boldsymbol{s}_n^{(k)}) = \underset{\boldsymbol{\ell}_n \in \mathcal{L}_n}{\arg\min} \sum_t \ell_{n,t} c_t(s_{n,t} + \ell_{n,t})$

end for

$\varepsilon^{(k)} = \left\| \boldsymbol{\ell}^{(k+1)} - \boldsymbol{\ell}^{(k)} \right\|$

end while

Two decentralized algorithms

Best Response (BR) Algorithm

Require: $\boldsymbol{\ell}^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},$

$k \leftarrow 0,\ \ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}$ & $k \leq k_{\max}$ **do**

for $n = 1$ to N **do**

$\boldsymbol{s}_n^{(k)} = \sum_{m<n} \boldsymbol{\ell}_m^{(k+1)} + \sum_{m>n} \boldsymbol{\ell}_m^{(k)}$

$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \mathrm{BR}_n(\boldsymbol{s}_n^{(k)}) = \underset{\boldsymbol{\ell}_n \in \mathcal{L}_n}{\operatorname{arg\,min}} \sum_t \ell_{n,t} c_t(s_{n,t} + \ell_{n,t})$

end for

$\varepsilon^{(k)} = \left\|\boldsymbol{\ell}^{(k+1)} - \boldsymbol{\ell}^{(k)}\right\|$

$k \leftarrow k+1$

end while

Two decentralized algorithms

Best Response (BR) Algorithm / Simultaneous Improving Response (SIR)

Require: $\boldsymbol{\ell}^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},\ \gamma$

$k \leftarrow 0,\ \ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}$ & $k \leq k_{\max}$ **do**

for $n = 1$ to N **do**

$$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \Pi_{\mathcal{L}_n}\left(\boldsymbol{\ell}_n^{(k)} - \gamma \nabla_n b_n(\boldsymbol{\ell}_n^{(k)}, \boldsymbol{\ell}_{-n}^{(k)})\right)$$

end for

$\varepsilon^{(k)} = \left\| \boldsymbol{\ell}^{(k+1)} - \boldsymbol{\ell}^{(k)} \right\|$

$k \leftarrow k+1$

end while

Two decentralized algorithms

Best Response (BR) Algorithm / Simultaneous Improving Response (SIR)

Require: $\boldsymbol{\ell}^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},\ \gamma$

$k \leftarrow 0,\ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}$ & $k \leq k_{\max}$ **do**

for $n = 1$ to N **do**

$\boldsymbol{s}_n^{(k)} = \sum_{m<n} \boldsymbol{\ell}_m^{(k+1)} + \sum_{m>n} \boldsymbol{\ell}_m^{(k)}$

$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \mathrm{BR}_n(\boldsymbol{s}_n^{(k)}) = \underset{\boldsymbol{\ell}_n \in \mathcal{L}_n}{\arg\min} \sum_t \ell_{n,t} c_t(s_{n,t} + \ell_{n,t})$

$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \Pi_{\mathcal{L}_n} \left(\boldsymbol{\ell}_n^{(k)} - \gamma \nabla_n b_n(\boldsymbol{\ell}_n^{(k)}, \boldsymbol{\ell}_{-n}^{(k)}) \right)$

end for

$\varepsilon^{(k)} = \left\| \boldsymbol{\ell}^{(k+1)} - \boldsymbol{\ell}^{(k)} \right\|$

$k \leftarrow k + 1$

end while

Two decentralized algorithms

Best Response (BR) Algorithm / Simultaneous Improving Response (SIR)

Require: $\boldsymbol{\ell}^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},\ \gamma$

$k \leftarrow 0,\ \ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}$ & $k \leq k_{\max}$ **do**

for $n = 1$ to N **do**

$\boldsymbol{s}_n^{(k)} = \sum_{m<n} \boldsymbol{\ell}_m^{(k+1)} + \sum_{m>n} \boldsymbol{\ell}_m^{(k)}$

$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \mathrm{BR}_n(\boldsymbol{s}_n^{(k)}) = \underset{\boldsymbol{\ell}_n \in \mathcal{L}_n}{\arg\min} \sum_t \ell_{n,t} c_t(s_{n,t} + \ell_{n,t})$

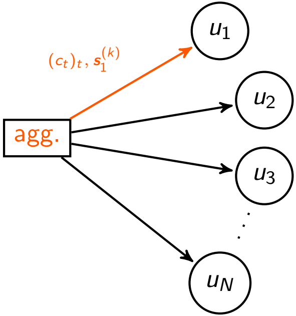
$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \Pi_{\mathcal{L}_n}\left(\boldsymbol{\ell}_n^{(k)} - \gamma \nabla_n b_n(\boldsymbol{\ell}_n^{(k)}, \boldsymbol{\ell}_{-n}^{(k)})\right)$

end for

$\varepsilon^{(k)} = \left\| \boldsymbol{\ell}^{(k+1)} - \boldsymbol{\ell}^{(k)} \right\|$

$k \leftarrow k + 1$

end while



Two decentralized algorithms

Best Response (BR) Algorithm / Simultaneous Improving Response (SIR)

Require: $\boldsymbol{\ell}^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},\ \gamma$

$k \leftarrow 0,\ \ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}\ \&\ k \leq k_{\max}$ **do**

for $n = 1$ to N **do**

$\boldsymbol{s}_n^{(k)} = \sum_{m<n} \boldsymbol{\ell}_m^{(k+1)} + \sum_{m>n} \boldsymbol{\ell}_m^{(k)}$

$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \mathrm{BR}_n(\boldsymbol{s}_n^{(k)}) = \underset{\boldsymbol{\ell}_n \in \mathcal{L}_n}{\arg\min} \sum_t \ell_{n,t} c_t(s_{n,t} + \ell_{n,t})$

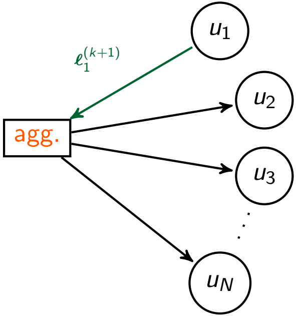
$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \Pi_{\mathcal{L}_n}\left(\boldsymbol{\ell}_n^{(k)} - \gamma \nabla_n b_n(\boldsymbol{\ell}_n^{(k)}, \boldsymbol{\ell}_{-n}^{(k)})\right)$

end for

$\varepsilon^{(k)} = \left\|\boldsymbol{\ell}^{(k+1)} - \boldsymbol{\ell}^{(k)}\right\|$

$k \leftarrow k+1$

end while



Two decentralized algorithms

Best Response (BR) Algorithm / Simultaneous Improving Response (SIR)

Require: $\boldsymbol{\ell}^{(0)}, \ k_{\max}, \ \varepsilon_{\text{stop}}, \ \gamma$

$k \leftarrow 0, \ \ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}$ & $k \leq k_{\max}$ **do**

for $n = 1$ to N **do**

$\boldsymbol{s}_n^{(k)} = \sum_{m<n} \boldsymbol{\ell}_m^{(k+1)} + \sum_{m>n} \boldsymbol{\ell}_m^{(k)}$

$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \mathrm{BR}_n(\boldsymbol{s}_n^{(k)}) = \underset{\boldsymbol{\ell}_n \in \mathcal{L}_n}{\arg\min} \sum_t \ell_{n,t} c_t(s_{n,t} + \ell_{n,t})$

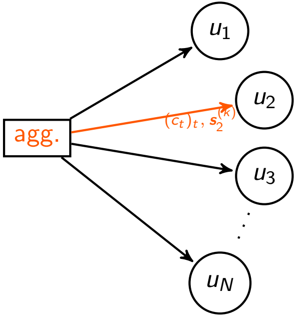
$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \Pi_{\mathcal{L}_n} \left(\boldsymbol{\ell}_n^{(k)} - \gamma \nabla_n b_n(\boldsymbol{\ell}_n^{(k)}, \boldsymbol{\ell}_{-n}^{(k)}) \right)$

end for

$\varepsilon^{(k)} = \left\| \boldsymbol{\ell}^{(k+1)} - \boldsymbol{\ell}^{(k)} \right\|$

$k \leftarrow k + 1$

end while



Two decentralized algorithms

Best Response (BR) Algorithm / Simultaneous Improving Response (SIR)

Require: $\boldsymbol{\ell}^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},\ \gamma$

$k \leftarrow 0,\ \ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}$ & $k \leq k_{\max}$ **do**

for $n = 1$ to N **do**

$\boldsymbol{s}_n^{(k)} = \sum_{m<n} \boldsymbol{\ell}_m^{(k+1)} + \sum_{m>n} \boldsymbol{\ell}_m^{(k)}$

$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \mathrm{BR}_n(\boldsymbol{s}_n^{(k)}) = \underset{\boldsymbol{\ell}_n \in \mathcal{L}_n}{\arg\min} \sum_t \ell_{n,t} c_t(s_{n,t} + \ell_{n,t})$

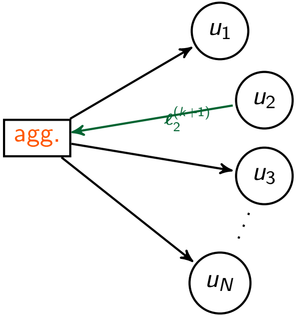
$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \Pi_{\mathcal{L}_n}\left(\boldsymbol{\ell}_n^{(k)} - \gamma \nabla_n b_n(\boldsymbol{\ell}_n^{(k)}, \boldsymbol{\ell}_{-n}^{(k)})\right)$

end for

$\varepsilon^{(k)} = \left\|\boldsymbol{\ell}^{(k+1)} - \boldsymbol{\ell}^{(k)}\right\|$

$k \leftarrow k + 1$

end while



Two decentralized algorithms

Best Response (BR) Algorithm / Simultaneous Improving Response (SIR)

Require: $\boldsymbol{\ell}^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},\ \gamma$

$k \leftarrow 0,\ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}\ \&\ k \leq k_{\max}$ **do**

for $n = 1$ to N **do**

$\boldsymbol{s}_n^{(k)} = \sum_{m<n} \boldsymbol{\ell}_m^{(k+1)} + \sum_{m>n} \boldsymbol{\ell}_m^{(k)}$

$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \mathrm{BR}_n(\boldsymbol{s}_n^{(k)}) = \operatorname*{arg\,min}_{\boldsymbol{\ell}_n \in \mathcal{L}_n} \sum_t \ell_{n,t} c_t(s_{n,t} + \ell_{n,t})$

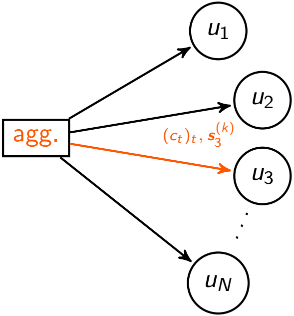
$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \Pi_{\mathcal{L}_n}\left(\boldsymbol{\ell}_n^{(k)} - \gamma \nabla_n b_n(\boldsymbol{\ell}_n^{(k)}, \boldsymbol{\ell}_{-n}^{(k)})\right)$

end for

$\varepsilon^{(k)} = \left\| \boldsymbol{\ell}^{(k+1)} - \boldsymbol{\ell}^{(k)} \right\|$

$k \leftarrow k + 1$

end while



Two decentralized algorithms

Best Response (BR) Algorithm / Simultaneous Improving Response (SIR)

Require: $\boldsymbol{\ell}^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},\ \gamma$

$k \leftarrow 0,\ \ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}\ \&\ k \leq k_{\max}$ **do**

for $n = 1$ to N **do**

$\boldsymbol{s}_n^{(k)} = \sum_{m<n} \boldsymbol{\ell}_m^{(k+1)} + \sum_{m>n} \boldsymbol{\ell}_m^{(k)}$

$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \mathrm{BR}_n(\boldsymbol{s}_n^{(k)}) = \arg\min_{\boldsymbol{\ell}_n \in \mathcal{L}_n} \sum_t \ell_{n,t} c_t(s_{n,t} + \ell_{n,t})$

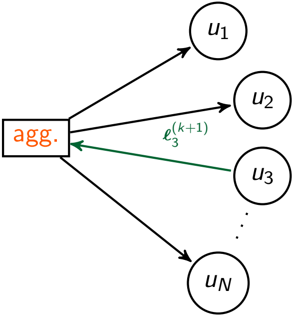
$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \Pi_{\mathcal{L}_n}\left(\boldsymbol{\ell}_n^{(k)} - \gamma \nabla_n b_n(\boldsymbol{\ell}_n^{(k)}, \boldsymbol{\ell}_{-n}^{(k)})\right)$

end for

$\varepsilon^{(k)} = \left\|\boldsymbol{\ell}^{(k+1)} - \boldsymbol{\ell}^{(k)}\right\|$

$k \leftarrow k + 1$

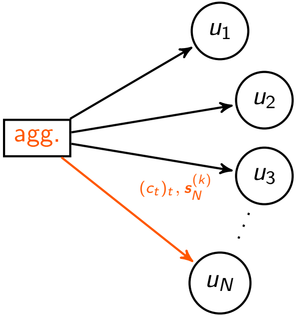
end while



Two decentralized algorithms

Best Response (BR) Algorithm / Simultaneous Improving Response (SIR)

Require: $\boldsymbol{\ell}^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},\ \gamma$
$k \leftarrow 0,\ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$
while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}\ \&\ k \leq k_{\max}$ **do**
for $n = 1$ to N **do**
$\boldsymbol{s}_n^{(k)} = \sum_{m<n} \boldsymbol{\ell}_m^{(k+1)} + \sum_{m>n} \boldsymbol{\ell}_m^{(k)}$
$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \mathrm{BR}_n(\boldsymbol{s}_n^{(k)}) = \operatorname{argmin}_{\boldsymbol{\ell}_n \in \mathcal{L}_n} \sum_t \ell_{n,t} c_t(s_{n,t} + \ell_{n,t})$
$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \Pi_{\mathcal{L}_n}\left(\boldsymbol{\ell}_n^{(k)} - \gamma \nabla_n b_n(\boldsymbol{\ell}_n^{(k)}, \boldsymbol{\ell}_{-n}^{(k)})\right)$
end for
$\varepsilon^{(k)} = \left\|\boldsymbol{\ell}^{(k+1)} - \boldsymbol{\ell}^{(k)}\right\|$
$k \leftarrow k+1$
end while



Two decentralized algorithms

Best Response (BR) Algorithm / Simultaneous Improving Response (SIR)

Require: $\boldsymbol{\ell}^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},\ \gamma$

$k \leftarrow 0,\ \ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}$ & $k \leq k_{\max}$ **do**

for $n = 1$ to N **do**

$\boldsymbol{s}_n^{(k)} = \sum_{m<n} \boldsymbol{\ell}_m^{(k+1)} + \sum_{m>n} \boldsymbol{\ell}_m^{(k)}$

$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \mathrm{BR}_n(\boldsymbol{s}_n^{(k)}) = \underset{\boldsymbol{\ell}_n \in \mathcal{L}_n}{\arg\min} \sum_t \ell_{n,t} c_t(s_{n,t} + \ell_{n,t})$

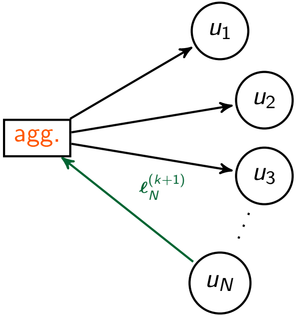
$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \Pi_{\mathcal{L}_n}\left(\boldsymbol{\ell}_n^{(k)} - \gamma \nabla_n b_n(\boldsymbol{\ell}_n^{(k)}, \boldsymbol{\ell}_{-n}^{(k)})\right)$

end for

$\varepsilon^{(k)} = \left\| \boldsymbol{\ell}^{(k+1)} - \boldsymbol{\ell}^{(k)} \right\|$

$k \leftarrow k + 1$

end while



Two decentralized algorithms

Best Response (BR) Algorithm / Simultaneous Improving Response (SIR)

Require: $\boldsymbol{\ell}^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},\ \gamma$

$k \leftarrow 0,\ \ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}\ \&\ k \leq k_{\max}$ **do**

for $n = 1$ to N **do**

$\boldsymbol{s}_n^{(k)} = \sum_{m<n} \boldsymbol{\ell}_m^{(k+1)} + \sum_{m>n} \boldsymbol{\ell}_m^{(k)}$

$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \mathrm{BR}_n(\boldsymbol{s}_n^{(k)}) = \underset{\boldsymbol{\ell}_n \in \mathcal{L}_n}{\arg\min} \sum_t \ell_{n,t} c_t(s_{n,t} + \ell_{n,t})$

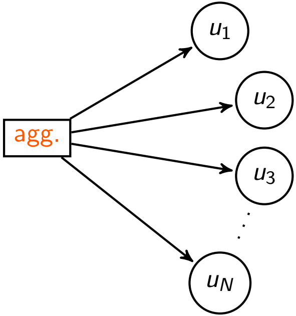
$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \Pi_{\mathcal{L}_n}\left(\boldsymbol{\ell}_n^{(k)} - \gamma \nabla_n b_n(\boldsymbol{\ell}_n^{(k)}, \boldsymbol{\ell}_{-n}^{(k)})\right)$

end for

$\varepsilon^{(k)} = \left\|\boldsymbol{\ell}^{(k+1)} - \boldsymbol{\ell}^{(k)}\right\|$

$k \leftarrow k + 1$

end while



Two decentralized algorithms

Best Response (BR) Algorithm / Simultaneous Improving Response (SIR)

Require: $\boldsymbol{\ell}^{(0)}, k_{\max}, \varepsilon_{\text{stop}}, \gamma$

$k \leftarrow 0, \ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}$ & $k \leq k_{\max}$ **do**

for $n = 1$ to N **do**

$\boldsymbol{s}_n^{(k)} = \sum_{m<n} \boldsymbol{\ell}_m^{(k+1)} + \sum_{m>n} \boldsymbol{\ell}_m^{(k)}$

$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \mathrm{BR}_n(\boldsymbol{s}_n^{(k)}) = \underset{\boldsymbol{\ell}_n \in \mathcal{L}_n}{\arg\min} \sum_t \ell_{n,t} c_t(s_{n,t} + \ell_{n,t})$

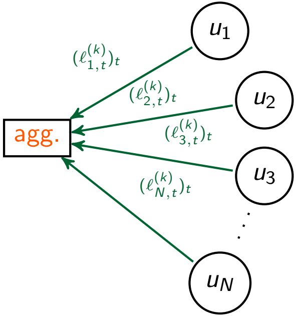
$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \Pi_{\mathcal{L}_n} \left(\boldsymbol{\ell}_n^{(k)} - \gamma \nabla_n b_n(\boldsymbol{\ell}_n^{(k)}, \boldsymbol{\ell}_{-n}^{(k)}) \right)$

end for

$\varepsilon^{(k)} = \left\| \boldsymbol{\ell}^{(k+1)} - \boldsymbol{\ell}^{(k)} \right\|$

$k \leftarrow k + 1$

end while



Two decentralized algorithms

Best Response (BR) Algorithm / Simultaneous Improving Response (SIR)

Require: $\boldsymbol{\ell}^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},\ \gamma$

$k \leftarrow 0,\ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}$ & $k \leq k_{\max}$ **do**

for $n = 1$ to N **do**

$\boldsymbol{s}_n^{(k)} = \sum_{m<n} \boldsymbol{\ell}_m^{(k+1)} + \sum_{m>n} \boldsymbol{\ell}_m^{(k)}$

$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \mathrm{BR}_n(\boldsymbol{s}_n^{(k)}) = \underset{\boldsymbol{\ell}_n \in \mathcal{L}_n}{\arg\min} \sum_t \ell_{n,t} c_t(s_{n,t} + \ell_{n,t})$

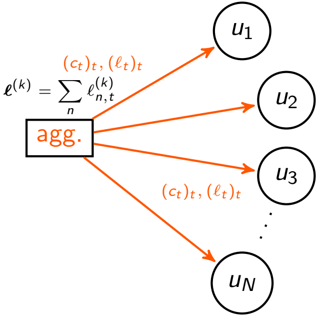
$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \Pi_{\mathcal{L}_n}\left(\boldsymbol{\ell}_n^{(k)} - \gamma \nabla_n b_n(\boldsymbol{\ell}_n^{(k)}, \boldsymbol{\ell}_{-n}^{(k)})\right)$

end for

$\varepsilon^{(k)} = \left\|\boldsymbol{\ell}^{(k+1)} - \boldsymbol{\ell}^{(k)}\right\|$

$k \leftarrow k + 1$

end while



Two decentralized algorithms

Best Response (BR) Algorithm / Simultaneous Improving Response (SIR)

Require: $\boldsymbol{\ell}^{(0)},\ k_{\max},\ \varepsilon_{\text{stop}},\ \gamma$

$k \leftarrow 0,\ \varepsilon^{(0)} \leftarrow \varepsilon_{\text{stop}}$

while $\varepsilon^{(k)} \geq \varepsilon_{\text{stop}}$ & $k \leq k_{\max}$ **do**

for $n = 1$ to N **do**

$\boldsymbol{s}_n^{(k)} = \sum_{m<n} \boldsymbol{\ell}_m^{(k+1)} + \sum_{m>n} \boldsymbol{\ell}_m^{(k)}$

$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \mathrm{BR}_n(\boldsymbol{s}_n^{(k)}) = \underset{\boldsymbol{\ell}_n \in \mathcal{L}_n}{\arg\min} \sum_t \ell_{n,t} c_t(s_{n,t} + \ell_{n,t})$

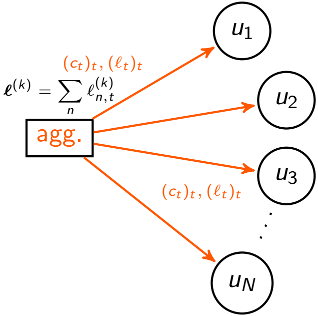
$\boldsymbol{\ell}_n^{(k+1)} \leftarrow \Pi_{\mathcal{L}_n}\left(\boldsymbol{\ell}_n^{(k)} - \gamma \nabla_n b_n(\boldsymbol{\ell}_n^{(k)}, \boldsymbol{\ell}_{-n}^{(k)})\right)$

end for

$\varepsilon^{(k)} = \left\|\boldsymbol{\ell}^{(k+1)} - \boldsymbol{\ell}^{(k)}\right\|$

$k \leftarrow k + 1$

end while



rm: sequential/simultaneous: SIR can be parallelized, but not BR (divergence)!

Fast Convergence Results

Theorem (Convergence of BR)

With aff. prices, BR cvg to $\boldsymbol{\ell}^{\mathrm{NE}}$ *with:*

$$\left\|\boldsymbol{\ell}^{(k)} - \boldsymbol{\ell}^{\mathrm{NE}}\right\|_2 \leq CN \times \frac{1}{\sqrt{k}}$$

Fast Convergence Results

Theorem (Convergence of BR)

With aff. prices, BR cvg to $\boldsymbol{\ell}^{\mathrm{NE}}$ with:

$$\left\|\boldsymbol{\ell}^{(k)} - \boldsymbol{\ell}^{\mathrm{NE}}\right\|_2 \leq CN \times \frac{1}{\sqrt{k}}$$

Theorem (Convergence of SIR)

With strong monot., SIR cvg with:

$$\left\|\boldsymbol{\ell}^{\mathrm{NE}} - \boldsymbol{\ell}^{(k)}\right\|_2 < (1-\frac{a^2}{NM^2})^k \left\|\boldsymbol{\ell}^{\mathrm{NE}} - \boldsymbol{\ell}^{(0)}\right\|_2$$

Fast Convergence Results

Theorem (Convergence of BR)

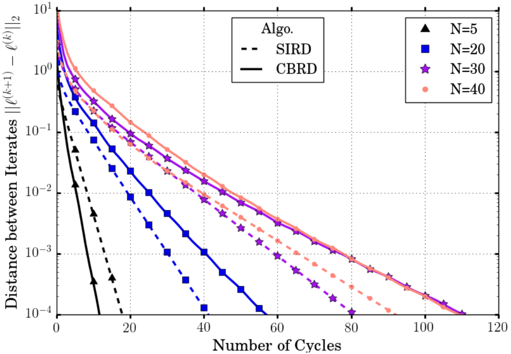
With aff. prices, BR cvg to ℓ^{NE} *with:*

$$\left\|\ell^{(k)} - \ell^{\mathrm{NE}}\right\|_2 \leq CN \times \frac{1}{\sqrt{k}}$$

Theorem (Convergence of SIR)

With strong monot., SIR cvg with:

$$\left\|\ell^{\mathrm{NE}} - \ell^{(k)}\right\|_2 < (1 - \frac{a^2}{NM^2})^k \left\|\ell^{\mathrm{NE}} - \ell^{(0)}\right\|_2$$



How complex are subproblems?

To solve the projection on $\mathcal{L}_n$ / to solve BR_n:

- for constraints $\mathcal{L}_n = \{\sum_{t\in\mathcal{T}} \ell_{n,t} = E_n \ , \quad \underline{\ell}_{n,t} \leq \ell_{n,t} \leq \overline{\ell}_{n,t}, \forall t \in \mathcal{T}\}$ (and with affine prices) $\rightarrow \mathcal{O}(T)$ algo.

How complex are subproblems?

To solve the projection on $\mathcal{L}_n$ / to solve BR_n:

- for constraints $\mathcal{L}_n = \{\sum_{t\in\mathcal{T}} \ell_{n,t} = E_n \ , \quad \underline{\ell}_{n,t} \leq \ell_{n,t} \leq \overline{\ell}_{n,t}, \forall t \in \mathcal{T}\}$ (and with affine prices) $\rightarrow \mathcal{O}(T)$ algo.
- for $\mathcal{L}_n$ a polytope $\rightarrow$ QP $\rightarrow$ something like $\mathcal{O}(T^{3.5})$...

How complex are subproblems?

To solve the projection on $\mathcal{L}_n$ / to solve BR_n:

- for constraints $\mathcal{L}_n = \{\sum_{t\in\mathcal{T}} \ell_{n,t} = E_n \ , \quad \underline{\ell}_{n,t} \leq \ell_{n,t} \leq \overline{\ell}_{n,t}, \forall t \in \mathcal{T}\}$ (and with affine prices) $\rightarrow \mathcal{O}(T)$ algo.
- for $\mathcal{L}_n$ a polytope $\rightarrow$ QP $\rightarrow$ something like $\mathcal{O}(T^{3.5})$...
- for $\mathcal{L}_n$ a general compact convex and general convex prices: ...

How complex are subproblems?

To solve the projection on $\mathcal{L}_n$ / to solve BR_n:

- for constraints $\mathcal{L}_n = \{\sum_{t\in\mathcal{T}} \ell_{n,t} = E_n \ , \quad \underline{\ell}_{n,t} \leq \ell_{n,t} \leq \overline{\ell}_{n,t}, \forall t \in \mathcal{T}\}$ (and with affine prices) $\rightarrow \mathcal{O}(T)$ algo.
- for $\mathcal{L}_n$ a polytope $\rightarrow$ QP $\rightarrow$ something like $\mathcal{O}(T^{3.5})$...
- for $\mathcal{L}_n$ a general compact convex and general convex prices: ...
- for $\mathcal{L}_n$ nonconvex (e.g. discrete power levels or block constraints): long...

How complex are subproblems?

To solve the projection on $\mathcal{L}_n$ / to solve BR_n:

- for constraints $\mathcal{L}_n = \{\sum_{t\in\mathcal{T}} \ell_{n,t} = E_n \ , \quad \underline{\ell}_{n,t} \leq \ell_{n,t} \leq \overline{\ell}_{n,t}, \forall t \in \mathcal{T}\}$ (and with affine prices) $\rightarrow \mathcal{O}(T)$ algo.
- for $\mathcal{L}_n$ a polytope $\rightarrow$ QP $\rightarrow$ something like $\mathcal{O}(T^{3.5})$...
- for $\mathcal{L}_n$ a general compact convex and general convex prices: ...
- for $\mathcal{L}_n$ nonconvex (e.g. discrete power levels or block constraints): long...

Not every model is scalable to small time steps!!

Online versus Offline procedure

- the coordination need to be done before real time: for instance the NE is computed at 11AM for the next 24 hours,

Online versus Offline procedure

- the coordination need to be done before real time: for instance the NE is computed at 11AM for the next 24 hours,
- the coordination depends on several parameters,

Online versus Offline procedure

- the coordination need to be done before real time: for instance the NE is computed at 11AM for the next 24 hours,
- the coordination depends on several parameters,
- in particular, the price signals c_t depend on the aggregated nonflexible consumption profile D_t

Online versus Offline procedure

- the coordination need to be done before real time: for instance the NE is computed at 11AM for the next 24 hours,
- the coordination depends on several parameters,
- in particular, the price signals c_t depend on the aggregated nonflexible consumption profile D_t
- those parameters need to be forecast in advance to compute the NE

Online versus Offline procedure

- the coordination need to be done before real time: for instance the NE is computed at 11AM for the next 24 hours,
- the coordination depends on several parameters,
- in particular, the price signals c_t depend on the aggregated nonflexible consumption profile D_t
- those parameters need to be forecast in advance to compute the NE
- to minimize forecast errors and take updates, we consider an online procedure with "receding horizons"

Online procedure

Start at $t = 1$

Online procedure

Start at $t = 1$

while $t \leq T$ **do**

Online procedure

Start at $t = 1$

while $t \leq T$ **do**

Set new horizon $\mathcal{T}^{(t)} = \{t, t+1, \ldots, T\}$

Online procedure

Start at $t = 1$

while $t \leq T$ **do**

Set new horizon $\mathcal{T}^{(t)} = \{t, t+1, \ldots, T\}$

Get $\boldsymbol{D}$ forecast on $\mathcal{T}^{(t)}$: $\hat{\boldsymbol{D}}^{(t)} \overset{\text{def}}{=} \left(\hat{D}^{(t)}{}_s\right)_{t \leq s \leq T}$

Online procedure

Start at $t = 1$
while $t \leq T$ **do**
Set new horizon $\mathcal{T}^{(t)} = \{t, t+1, \dots, T\}$
Get $\boldsymbol{D}$ forecast on $\mathcal{T}^{(t)}$: $\hat{\boldsymbol{D}}^{(t)} \stackrel{\text{def}}{=} \left(\hat{D}^{(t)}{}_s\right)_{t \leq s \leq T}$
Re-compute prices $c_t(.)$ for $t \in \mathcal{T}^{(t)}$ with $\hat{\boldsymbol{D}}$

Online procedure

Start at $t = 1$

while $t \leq T$ **do**

Set new horizon $\mathcal{T}^{(t)} = \{t, t+1, \dots, T\}$

Get $\boldsymbol{D}$ forecast on $\mathcal{T}^{(t)}$: $\hat{\boldsymbol{D}}^{(t)} \overset{\text{def}}{=} \left(\hat{D}^{(t)}{}_s\right)_{t \leq s \leq T}$

Re-compute prices $c_t(.)$ for $t \in \mathcal{T}^{(t)}$ with $\hat{\boldsymbol{D}}$

Run Algo. SIRD or BRD to compute NE $\ell^{(t)}$ on $\mathcal{T}^{(t)}$

Online procedure

Start at $t = 1$

while $t \leq T$ **do**

Set new horizon $\mathcal{T}^{(t)} = \{t, t+1, \dots, T\}$

Get $\boldsymbol{D}$ forecast on $\mathcal{T}^{(t)}$: $\hat{\boldsymbol{D}}^{(t)} \stackrel{\text{def}}{=} \left(\hat{D}^{(t)}{}_s\right)_{t \leq s \leq T}$

Re-compute prices $c_t(.)$ for $t \in \mathcal{T}^{(t)}$ with $\hat{\boldsymbol{D}}$

Run Algo. SIRD or BRD to compute NE $\ell^{(t)}$ on $\mathcal{T}^{(t)}$

for each user $n \in \mathcal{N}$ **do**

Online procedure

Start at $t = 1$
while $t \leq T$ **do**
  Set new horizon $\mathcal{T}^{(t)} = \{t, t+1, \dots, T\}$
  Get $\boldsymbol{D}$ forecast on $\mathcal{T}^{(t)}$: $\hat{\boldsymbol{D}}^{(t)} \stackrel{\text{def}}{=} \left(\hat{D}^{(t)}{}_s\right)_{t \leq s \leq T}$
  Re-compute prices $c_t(.)$ for $t \in \mathcal{T}^{(t)}$ with $\hat{\boldsymbol{D}}$
  Run Algo. SIRD or BRD to compute NE $\boldsymbol{\ell}^{(t)}$ on $\mathcal{T}^{(t)}$
  for each user $n \in \mathcal{N}$ **do**
    Realize computed profile on time t, $\ell_{n,t}^{(t)}$

Online procedure

Start at $t = 1$

while $t \leq T$ **do**

Set new horizon $\mathcal{T}^{(t)} = \{t, t+1, \ldots, T\}$

Get $\boldsymbol{D}$ forecast on $\mathcal{T}^{(t)}$: $\hat{\boldsymbol{D}}^{(t)} \stackrel{\text{def}}{=} \left(\hat{D}^{(t)}{}_s\right)_{t \leq s \leq T}$

Re-compute prices $c_t(.)$ for $t \in \mathcal{T}^{(t)}$ with $\hat{\boldsymbol{D}}$

Run Algo. SIRD or BRD to compute NE $\boldsymbol{\ell}^{(t)}$ on $\mathcal{T}^{(t)}$

for each user $n \in \mathcal{N}$ **do**

Realize computed profile on time t, $\ell_{n,t}^{(t)}$

Update energy demand $E_n \leftarrow E_n - \ell_{n,t}^{(t)}$

Online procedure

Start at $t = 1$

while $t \leq T$ **do**

Set new horizon $\mathcal{T}^{(t)} = \{t, t+1, \dots, T\}$

Get $\boldsymbol{D}$ forecast on $\mathcal{T}^{(t)}$: $\hat{\boldsymbol{D}}^{(t)} \stackrel{\text{def}}{=} \left(\hat{D}^{(t)}{}_s\right)_{t \leq s \leq T}$

Re-compute prices $c_t(.)$ for $t \in \mathcal{T}^{(t)}$ with $\hat{\boldsymbol{D}}$

Run Algo. SIRD or BRD to compute NE $\boldsymbol{\ell}^{(t)}$ on $\mathcal{T}^{(t)}$

for each user $n \in \mathcal{N}$ **do**

Realize computed profile on time t, $\ell_{n,t}^{(t)}$

Update energy demand $E_n \leftarrow E_n - \ell_{n,t}^{(t)}$

end for

Online procedure

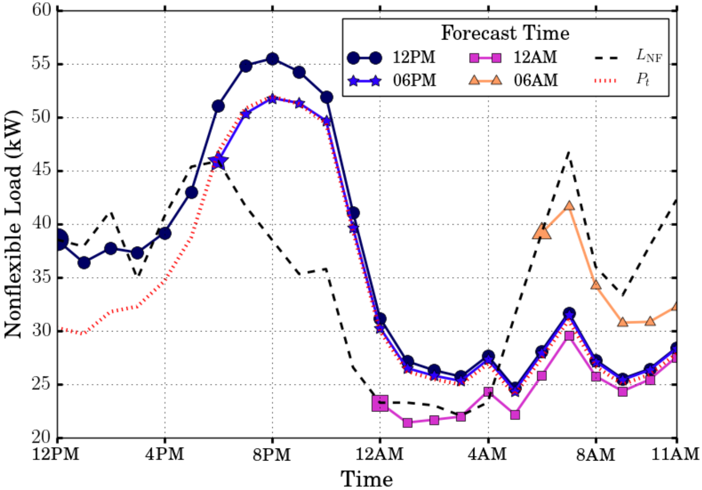
Start at $t = 1$
while $t \leq T$ **do**
  Set new horizon $\mathcal{T}^{(t)} = \{t, t+1, \dots, T\}$
  Get $\boldsymbol{D}$ forecast on $\mathcal{T}^{(t)}$: $\hat{\boldsymbol{D}}^{(t)} \stackrel{\text{def}}{=} \left(\hat{D}^{(t)}{}_s\right)_{t \leq s \leq T}$
  Re-compute prices $c_t(.)$ for $t \in \mathcal{T}^{(t)}$ with $\hat{\boldsymbol{D}}$
  Run Algo. SIRD or BRD to compute NE $\boldsymbol{\ell}^{(t)}$ on $\mathcal{T}^{(t)}$
  for each user $n \in \mathcal{N}$ **do**
    Realize computed profile on time t, $\ell_{n,t}^{(t)}$
    Update energy demand $E_n \leftarrow E_n - \ell_{n,t}^{(t)}$
  end for
  Wait for $t + 1$

Online procedure

Start at $t = 1$
while $t \leq T$ **do**
  Set new horizon $\mathcal{T}^{(t)} = \{t, t+1, \dots, T\}$
  Get $\boldsymbol{D}$ forecast on $\mathcal{T}^{(t)}$: $\hat{\boldsymbol{D}}^{(t)} \stackrel{\text{def}}{=} \left(\hat{D}^{(t)}{}_s\right)_{t \leq s \leq T}$
  Re-compute prices $c_t(.)$ for $t \in \mathcal{T}^{(t)}$ with $\hat{\boldsymbol{D}}$
  Run Algo. SIRD or BRD to compute NE $\boldsymbol{\ell}^{(t)}$ on $\mathcal{T}^{(t)}$
  for each user $n \in \mathcal{N}$ **do**
    Realize computed profile on time t, $\ell_{n,t}^{(t)}$
    Update energy demand $E_n \leftarrow E_n - \ell_{n,t}^{(t)}$
  end for
  Wait for $t + 1$
end while

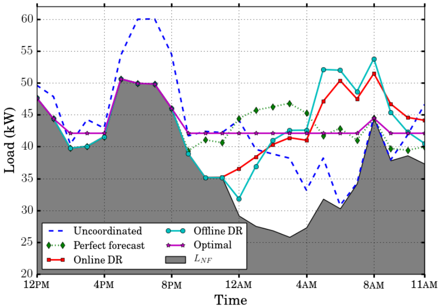
Forecasts of nonflexible Demand

Estimated with historical data



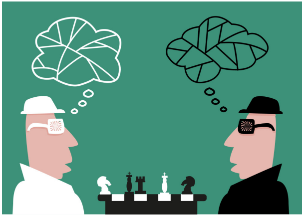
Online procedure achieves significant gains!

Cons. Scenario	Social Cost	Avg. Price	Gain
Uncoordinated	$ 1257.2	0.200 $/kWh	—
Offline DR	$ 1231.6	0.195 $/kWh	2.036%
Online DR	**$ 1131.1**	0.180 $/kWh	**10.03%**
Perfect forecast DR	$ 1075.2	0.171 $/kWh	14.47%
Optimal scenario	$ 1056.8	0.169 $/kWh	15.94%



Conclusion

- Game theoretic residential demand response might be worthy and efficient!



Conclusion

- Game theoretic residential demand response might be worthy and efficient!
- An online procedure will enable to limit the impact of forecast parameters

Conclusion

- Game theoretic residential demand response might be worthy and efficient!
- An online procedure will enable to limit the impact of forecast parameters

Several extensions to get closer to implementation:

Conclusion

- Game theoretic residential demand response might be worthy and efficient!
- An online procedure will enable to limit the impact of forecast parameters

Several extensions to get closer to implementation:

- other uncertain parameters: change in consumers constraints, disconnection, balancing market

Conclusion

- Game theoretic residential demand response might be worthy and efficient!
- An online procedure will enable to limit the impact of forecast parameters

Several extensions to get closer to implementation:

- other uncertain parameters: change in consumers constraints, disconnection, balancing market
- integration of distributed renewable generation,

Conclusion

- Game theoretic residential demand response might be worthy and efficient!
- An online procedure will enable to limit the impact of forecast parameters

Several extensions to get closer to implementation:

- other uncertain parameters: change in consumers constraints, disconnection, balancing market
- integration of distributed renewable generation,
- local network constraints on the distribution grid: may affect the equilibrium and convergence of algos!

Conclusion

- Game theoretic residential demand response might be worthy and efficient!
- An online procedure will enable to limit the impact of forecast parameters

Several extensions to get closer to implementation:

- other uncertain parameters: change in consumers constraints, disconnection, balancing market
- integration of distributed renewable generation,
- local network constraints on the distribution grid: may affect the equilibrium and convergence of algos!

THANK YOU!

Conclusion

- Game theoretic residential demand response might be worthy and efficient!
- An online procedure will enable to limit the impact of forecast parameters

Several extensions to get closer to implementation:

- other uncertain parameters: change in consumers constraints, disconnection, balancing market
- integration of distributed renewable generation,
- local network constraints on the distribution grid: may affect the equilibrium and convergence of algos!

[1] Jacquot, P., Beaude, O., Gaubert, S., and Oudjane, N. (2017a). Analysis and implementation of a hourly billing mechanism for demand response. *arXiv 1712.08622*.

[2] Jacquot, P., Beaude, O., Gaubert, S., and Oudjane, N. (2017b). Demand response in the smart grid: the impact of consumers temporal preferences. In *2017 IEEE Int. Conf. Smart Grid Comm. (SmartGridComm)*.

[3] Jacquot, P., Beaude, O., Gaubert, S., and Oudjane, N. (2017c). Demand side management in the smart grid: an efficiency and fairness tradeoff. IEEE.