Exploiting consumers flexibilities *via* a Demand Response mecanism: Decentralized Nash Computation

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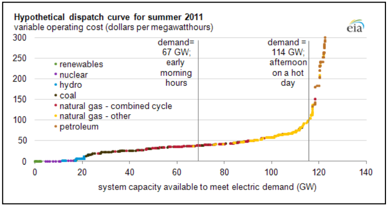
Réseau Optimisation
EDF Lab, Paris-Saclay

Introduction: Cost of Flexible Consumption



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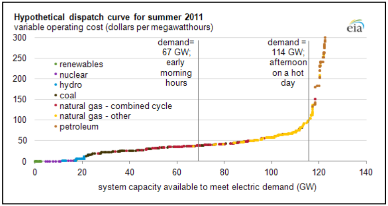

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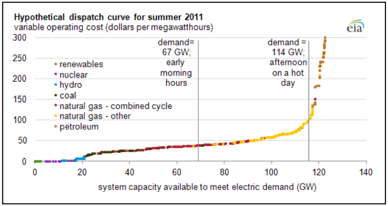

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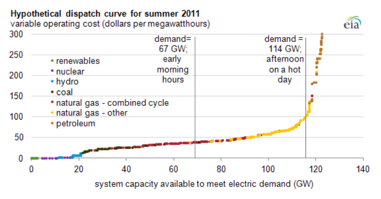

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$$\overline{C}_t(\ell^t), \ t = 1 \dots T$$

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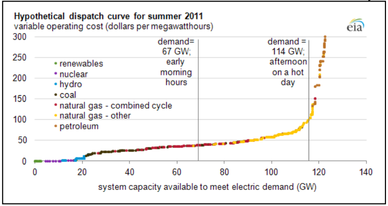

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- nonflexible consumption $(L_t)_t$
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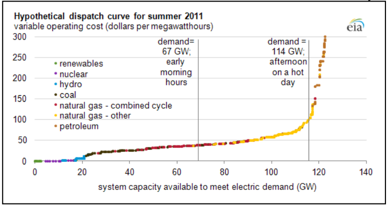

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Cost of flexible load:

$$C_t(\ell^t) := \overline{C}_t(L^t + \ell^t) - \overline{C}_t(L^t)\ .$$

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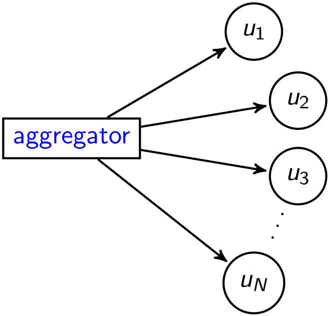

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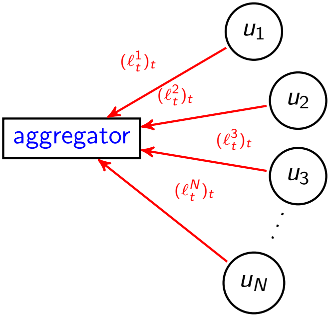

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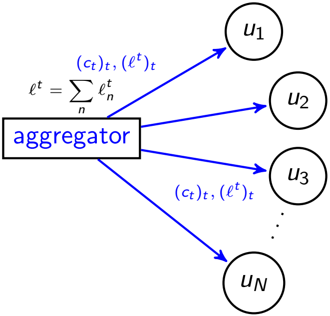

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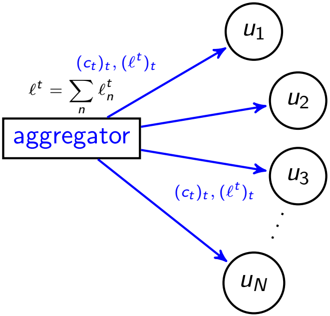

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- Consumers eventually reach an equilibrium.

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$\rightarrow$ N-person minimization game $\mathcal{G} := \left(\mathcal{N}, \mathcal{L}, (b_n)_n\right)$

Nash Equilibrium

$\rightarrow b_n(\boldsymbol{\ell}_n, \boldsymbol{\ell}_{-n})$ cost of consumer n

Definition

The profile $(\hat{\ell}_n)_n$ is a Nash Equilibrium *IFF* for all player n, for all possible admissible profile (strategy) $\boldsymbol{\ell}_n$:

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Equilibrium $\iff$ no one has any interest to change!

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1. **decentralized**: for privacy reasons and dimension of the global problem,
2. **asynchronous**: it would not be possible to synchronize efficiently if local optimizations are performed,
3. **fast**: the equilibrium may have to be recomputed.

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while $\left\|\boldsymbol{\ell}^{(k)} - \boldsymbol{\ell}^{(k+1)}\right\| > \varepsilon$ **do**

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$$\min_{\ell_n \in \mathcal{L}_n} \frac{E_n}{E} \sum_{t \in \mathcal{T}} C_t(\ell^t)$$

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Alternate Minimization :

Theorem (Hong et al., 2017)

Alternate minimization on convex function f over N blocks converges linearly. Precisely, after r cycles:

$$f(x^{(r)}) - \min_{x \in \mathcal{X}} f(x) \leq \; KN^2 \frac{1}{r} \; .$$

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$\Rightarrow$ Approximated $\varepsilon_r = \frac{\max E_n}{E} \frac{KN^2}{r}$ Nash Eq. in r cycles.

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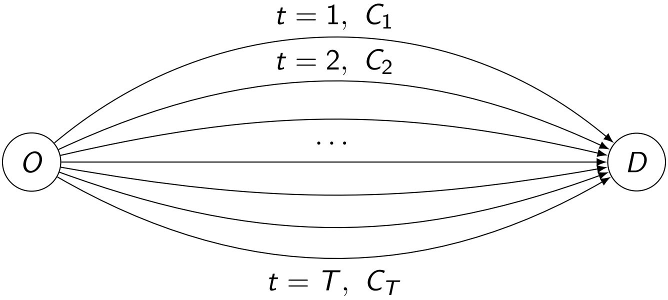

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In a network of parallel arcs with cost functions $\ell_n^t \mapsto \ell_n^t \times c_t(\ell^t)$, there exists a unique equilibrium.

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Proposition

The result extends to the constrained case $\underline{\ell_n^t} \leq \ell_n^t \leq \overline{\ell_n^t}$ *(and where each player has a subset of arcs).*

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where $\Phi(\boldsymbol{\ell}) = \sum_t \frac{a_t}{2} \left[(\ell^t)^2 + \sum_n (\ell_n^t)^2 \right] + b_t \ell^t$ is called a potential function:

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$$\iff \forall n,\ \forall \boldsymbol{\ell},\ \nabla_n b_n(\boldsymbol{\ell}) = \nabla_n \Phi(\boldsymbol{\ell})\ .$$

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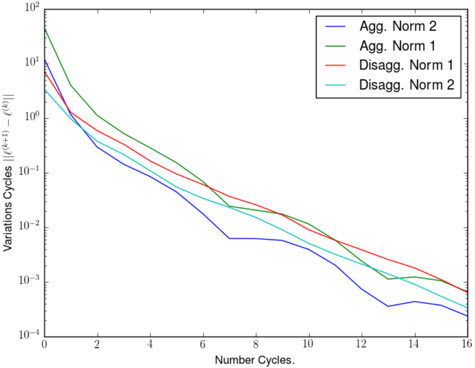

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The non-linear spectral radius of CBRD operator:

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Corrolary: the *BRD* converges with an exponential rate.

Bounding Efficiency

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... but how far from the optimal profile is it ?

Measuring Efficiency: the Price of Anarchy

Nash Equilibrium (NE)
$(\ell_n)_n$ is a NE *IFF* for all n:

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Definition (Price of Anarchy)

$$\mathrm{PoA}(\mathcal{G}) := \frac{\sup_{\ell \in \mathcal{X}_{\mathcal{G}}^{\mathrm{NE}}} \mathrm{SC}(\ell)}{\mathrm{SC}(\ell^*)} ,$$

where $\mathrm{SC}(.) = \sum_n b_n(.)$ is the *social cost*.

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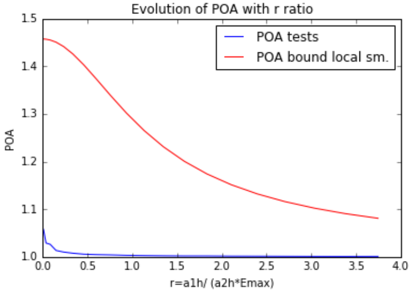

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With linear costs, if for all n and for all t , $\ell_n^t > 0$:

$$PoA = 1 + \frac{\left(1 - \frac{4N}{(N+1)^2}\right) V}{-V + 8\left(\sum_h \frac{\alpha_h}{\beta_h} E + E^2\right)} \tag{2}$$

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- Lower bound on the PoA...
- ... but no upper bound!
- Can we have some results in the nonlinear case ?

User's preferences

Consumers might have a prefered consumption profile $(\hat{\ell}_n^t)_t$

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What is the impact on the equilibrium profile and global system costs ?

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Proposition (Jacquot et al., 2017)

Assume $\forall n \in \mathcal{N}$, $\frac{\hat{\ell}_n^P}{E_n} + \frac{1}{2} \geq \frac{\hat{\ell}^P}{E}$ *, then, for* $\alpha \in (0,1]$*, the NE of* $\mathcal{G}_\alpha^{DP}$ *gives:*

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where $\phi(\alpha) \stackrel{def}{=} \frac{2\alpha}{(1+\alpha)+(1-\alpha)N} \in [0,1]$.

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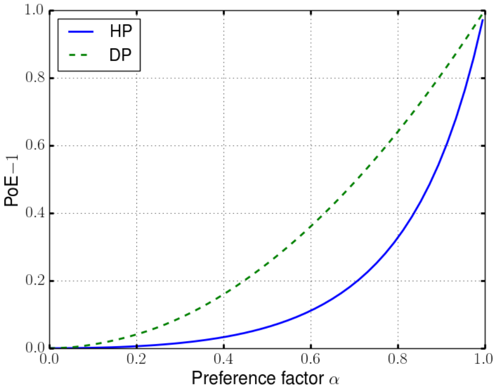

Evolution of PoE-1 with α.

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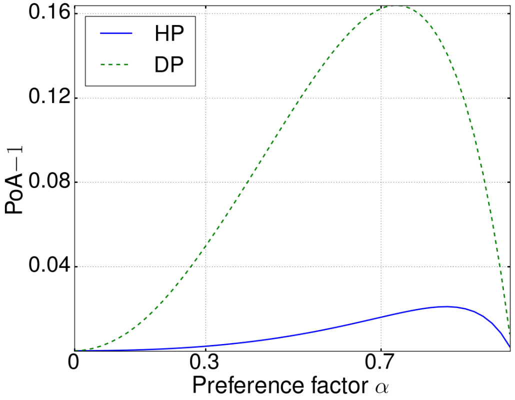

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THANK YOU!

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