# A Privacy-Preserving Disaggregation Algorithm for Nonconvex Optimization based on Alternate Projections

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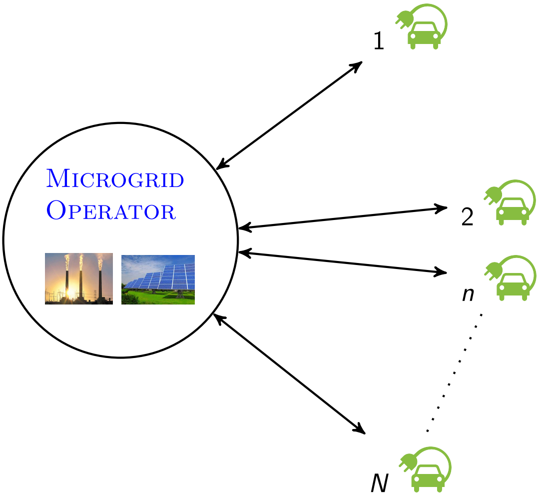
Journée de rentrée du **CMAP**

# Introduction and Context



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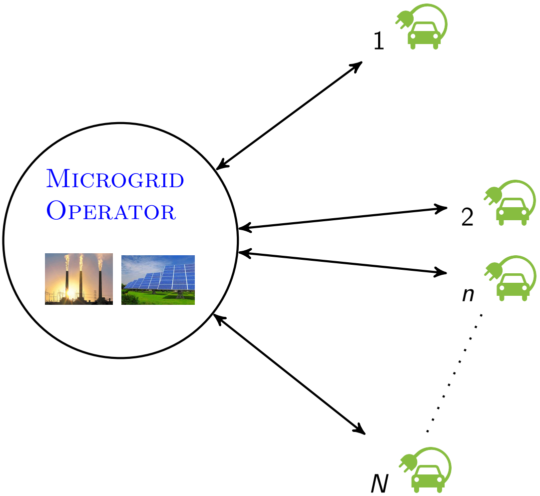

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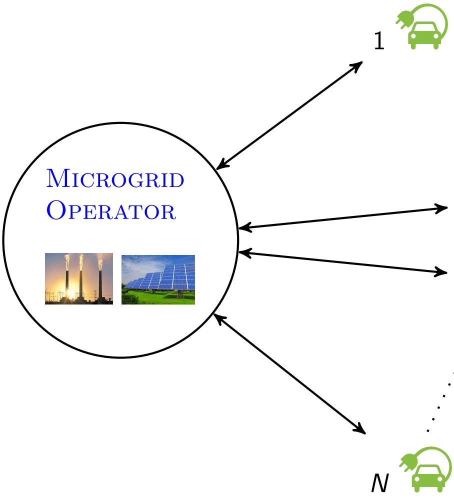

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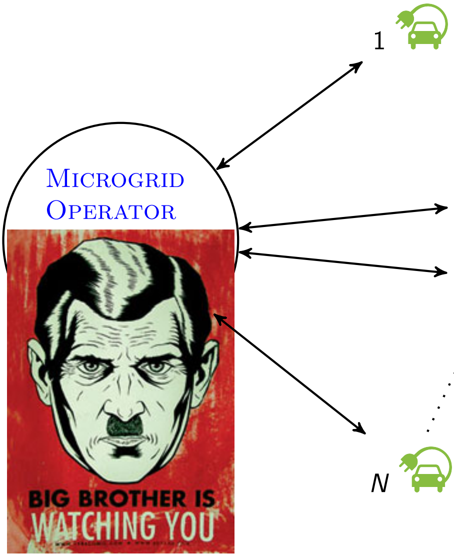
- **dimension**: hundreds or thousands of users/consumers ;
- **privacy**: users may not want to disclose **individual constraints** and **consumption profiles** to big brother.



# Problem Formulation

$$\min_{\boldsymbol{x}\in\mathbb{R}^{N\times T},\ \boldsymbol{p}\in\mathbb{R}^T} f(\boldsymbol{p}) \tag{1a}$$

$$\boldsymbol{p} \in \mathcal{P} \tag{1b}$$

$$\sum_{n\in\mathcal{N}} x_{n,t} = p_t,\ \forall t \in \mathcal{T} \tag{1c}$$

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with $\mathcal{X}_n \stackrel{\text{def}}{=} \left\{\boldsymbol{x}_n \in \mathbb{R}^T \mid \sum_t x_{n,t} = E_n \quad \text{and} \quad \forall t, \underline{x}_{n,t} \leq x_{n,t} \leq \overline{x}_{n,t}\right\}$

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RESSOURCE ALLOCATION PROBLEMS: many applications in energy, logistics, distributed computing, healthcare...

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- a lot of problems have non convex constraints/ cost functions : our method does not require convexity.

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**Disaggregation Problem**

$$\text{Find } \boldsymbol{x} = (\boldsymbol{x}_n)_{n\in\mathcal{N}} \in \mathcal{Y}_{\boldsymbol{p}} \cap \mathcal{X}$$

where $\mathcal{Y}_{\boldsymbol{p}} \stackrel{\text{def}}{=} \{\boldsymbol{y} \in \mathbb{R}^{NT} | \sum_{n\in\mathcal{N}} \boldsymbol{y}_n = \boldsymbol{p}\}$

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Characterizing $\mathcal{Y}_{\boldsymbol{p}} \cap \mathcal{X} = \{\boldsymbol{x} \in \mathcal{X} | \ \sum_{n\in\mathcal{N}} \boldsymbol{x}_n = \boldsymbol{p}\}$
Necessary aggregated constraints:

$$\sum_t p_t = \sum_n E_n \ \text{ and } \ \forall t, \ \sum_n \underline{x}_{n,t} \leq p_t \leq \sum_n \overline{x}_{n,t} \ .$$

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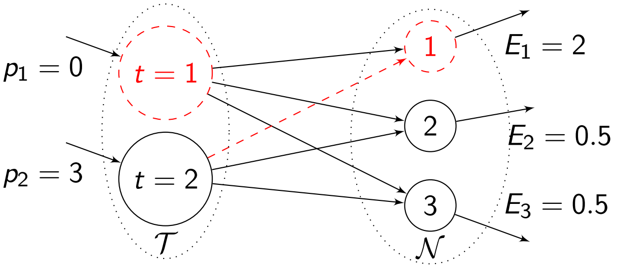

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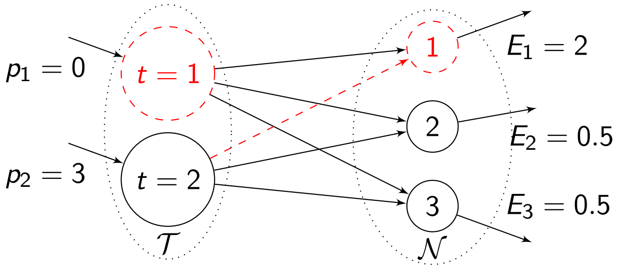
## Theorem (Hoffman Circulation's Theorem)

*Disaggregation is feasible (i.e. $\mathcal{X} \cap \mathcal{Y}_{\boldsymbol{p}} \neq \emptyset$) iff for any $\mathcal{T}_{\mathrm{in}} \subset \mathcal{T}, \mathcal{N}_{\mathrm{in}} \subset \mathcal{N}$:*

$$\sum_{t\notin\mathcal{T}_{\mathrm{in}}} p_t \leq \sum_{t\notin\mathcal{T}_{\mathrm{in}}, n\in\mathcal{N}_{\mathrm{in}}} \overline{x}_{n,t} - \sum_{t\in\mathcal{T}_{\mathrm{in}}, n\notin\mathcal{N}_{\mathrm{in}}} \underline{x}_{n,t} + \sum_{n\notin\mathcal{N}_{\mathrm{in}}} E_n. \quad (2)$$



# Alternate Projections Algorithm

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**Require:** $\boldsymbol{y}^{(0)}$, $k = 0$ , $\varepsilon_{\text{cvg}}$, $\|.\|$
**repeat**
$\boldsymbol{x}^{(k+1)} \leftarrow P_{\mathcal{X}}(\boldsymbol{y}^{(k)})$
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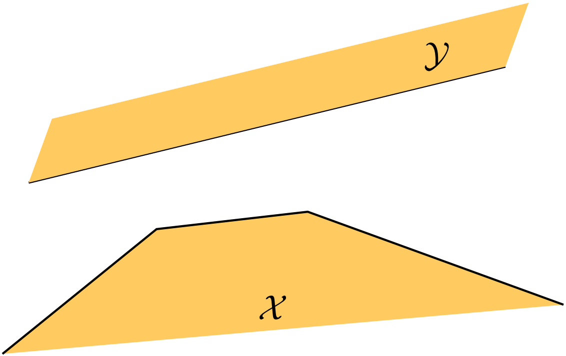
**until** $\left\|\boldsymbol{y}^{(k)} - \boldsymbol{y}^{(k-1)}\right\| < \varepsilon_{\text{cvg}}$



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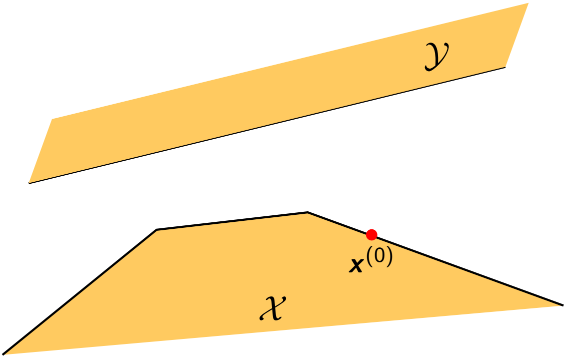

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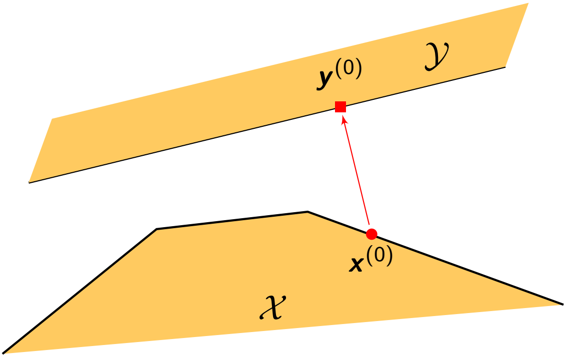

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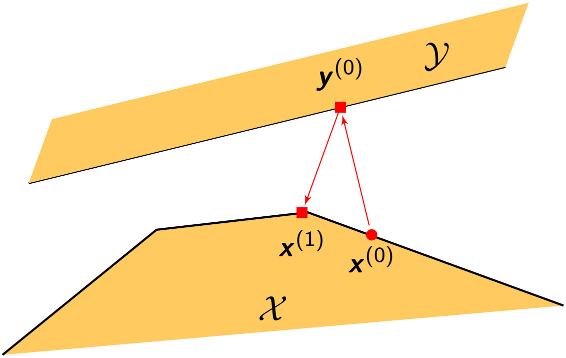

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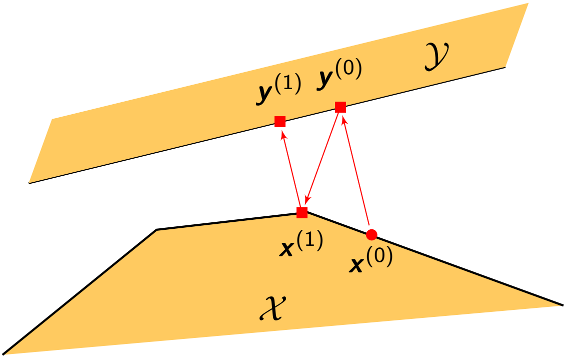

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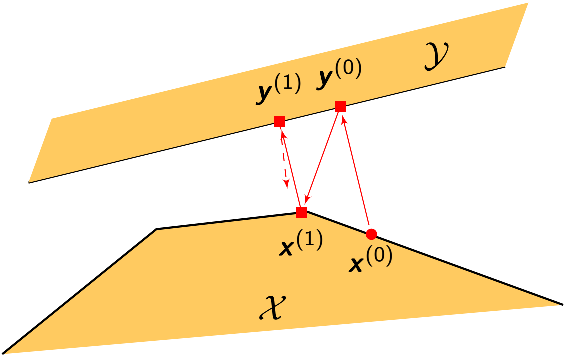

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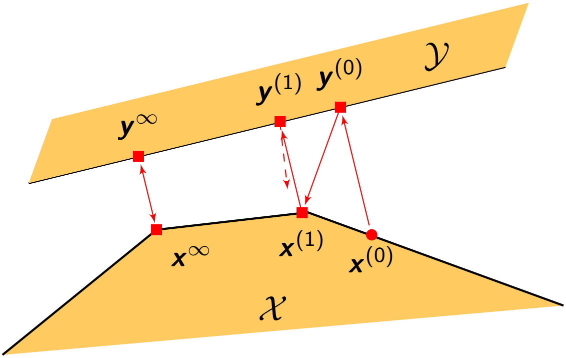

## Theorem (Gubin, Polyak, 1967)

*Let $\mathcal{X}$ and $\mathcal{Y}$ be two convex sets with $\mathcal{X}$ bounded, and let $(\boldsymbol{x}^{(k)})_k$ and $(\boldsymbol{y}^{(k)})_k$ be the two infinite sequences generated by APM with $\varepsilon_{cvg} = 0$. Then there exists $\boldsymbol{x}^\infty \in \mathcal{X}$ and $\boldsymbol{y}^\infty \in \mathcal{Y}$ such that:*

$$\boldsymbol{x}^{(k)} \underset{k\to\infty}{\longrightarrow} \boldsymbol{x}^\infty \ , \quad \boldsymbol{y}^{(k)} \underset{k\to\infty}{\longrightarrow} \boldsymbol{y}^\infty; \tag{3a}$$

$$\|\boldsymbol{x}^\infty - \boldsymbol{y}^\infty\|_2 = \min_{\boldsymbol{x}\in\mathcal{X},\boldsymbol{y}\in\mathcal{Y}} \|\boldsymbol{x} - \boldsymbol{y}\|_2 \ . \tag{3b}$$

*In particular, if $\mathcal{X} \cap \mathcal{Y} \neq \emptyset$, then $(\boldsymbol{x}^{(k)})_k$ and $(\boldsymbol{y}^{(k)})_k$ converge to a same point $\boldsymbol{x}^\infty \in \mathcal{X} \cap \mathcal{Y}$.*

## Theorem (Cut generation from APM limit iterates)

*For the sets $\mathcal{X}$ and $\mathcal{Y}$ defined above, and if $\mathcal{X} \cap \mathcal{Y} = \emptyset$ , the following sets given by the limit orbit $(\boldsymbol{x}^\infty, \boldsymbol{y}^\infty)$ defined in Theorem 2:*

(4a)

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*define a "Hoffman cut" violated by $\boldsymbol{p}$, that is:*

$$\sum_{n\in\mathcal{N}_0} E_n + \sum_{t\in\mathcal{T}_0, n\notin\mathcal{N}_0} \overline{x}_{n,t} - \sum_{t\notin\mathcal{T}_0, n\in\mathcal{N}_0} \underline{x}_{n,t} < \sum_{t\in\mathcal{T}_0} p_t \ . \tag{5}$$

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*This cut can be reformulated in terms of $\mathbb{1}_N^\top \boldsymbol{x}^\infty$ as:*

$$\mathrm{A}_{\mathcal{T}_0} < \sum_{t\in\mathcal{T}_0} p_t \ \textit{with}\ \mathrm{A}_{\mathcal{T}_0} \stackrel{\text{def}}{=} \sum_{t\in\mathcal{T}_0} \sum_{n\in\mathcal{N}} x^\infty_{n,t}. \tag{6}$$

# Linear convergence of APM in our case

**Theorem**

*For the sets $\mathcal{X}$ and $\mathcal{Y}$ defined above, the two subsequences of AP $(\boldsymbol{x}^{(k)})_k$ and $(\boldsymbol{y}^{(k)})_k$ converge at a geometric rate to $\boldsymbol{x}^\infty \in \mathcal{X}$, $\boldsymbol{y}^\infty \in \mathcal{Y}$, with:*

$$\|\boldsymbol{x}^{(k)} - \boldsymbol{x}^\infty\|_2 \leq 2\|\boldsymbol{x}^{(0)} - \boldsymbol{x}^\infty\|_2 \times \rho_{NT}^k$$

$$\text{where } \rho_{NT} \stackrel{\text{def}}{=} 1 - \frac{4}{N(T+1)^2(T-1)} < 1 \ ,$$

*Same inequalities hold for the convergence of $\boldsymbol{y}^{(k)}$ to $\boldsymbol{y}^\infty$.*

# Some Ingredients...

## Lemma (Nishihara et al, 2014)

*For APM on polyhedra $\mathcal{X}$ and $\mathcal{Y}$, the sequences $(\boldsymbol{x}^{(k)})_k$ and $(\boldsymbol{y}^{(k)})_k$ converge at a geometric rate, where the rate is bounded by the maximal value of the square of the* **cosine of the Friedrichs angle** *$c_F(U, V)$ between a face $U$ of $\mathcal{X}$ and a face $V$ of $\mathcal{Y}$, where $c_F(U, V)$ is given by:*

$$c_F(U, V) = \sup\{u^T v \mid \|u\| \leq 1, \|v\| \leq 1 \\ u \in U \cap (U \cap V)^\perp, v \in V \cap (U \cap V)^\perp\}.$$

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## Lemma (Nishihara et al, 2014)

*Let $A$ and $B$ be matrices with orthonormal rows and with equal numbers of columns and $\Lambda_{\mathrm{sv}}(AB^{\top})$ the set of singular values of $AB^{\top}$. Then:*

- *if $\Lambda_{\mathrm{sv}}(AB^{\top}) = \{1\}$, then $c_F(\mathrm{Ker(A)}, \mathrm{Ker(B)}) = 0$ ;*
- *Otherwise, $c_F(\mathrm{Ker(A)}, \mathrm{Ker(B)}) = \max_{\lambda<1}\{\lambda \in \Lambda_{\mathrm{sv}}(\mathrm{AB}^{\top})\}$.*

# Convergence rate: sketch of proof - 2:

- $\mathcal{Y}$ is affine subspace $\mathcal{Y} = \{\boldsymbol{x} \in \mathbb{R}^{NT} \,|A\boldsymbol{x} = \sqrt{N}^{-1}\mathbf{1}_T\}$ with $\overrightarrow{\mathcal{Y}} = \mathrm{Ker}(A)$ and $A \stackrel{\text{def}}{=} \sqrt{N}^{-1} J_{1,N} \otimes I_T$.

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- Faces of $\mathcal{X}$ are subsets of the collection of affine subspaces indexed by $(\overline{\mathcal{T}}_n, \underline{\mathcal{T}}_n)_n \subset \mathcal{T}^N$ (with $\overline{\mathcal{T}} \cap \underline{\mathcal{T}} = \emptyset$):

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- Denote $\bar{\mathcal{T}} \stackrel{\text{def}}{=} \cup_n \mathcal{T}_n^c$ and $P \stackrel{\text{def}}{=} I_T - S$. Then $P = \mathrm{diag}(P_{\bar{\mathcal{T}}}, 0_{\bar{\mathcal{T}}^c})$
$\rightarrow$ restrict to $\mathrm{Vect}(e_t)_{t\in\bar{\mathcal{T}}}$ to find $\lambda_1(P)$ (least positive eigval)

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- Consider graph $\mathcal{G} = (\bar{\mathcal{T}}, \mathcal{E})$ whose vertices set is $\bar{\mathcal{T}}$ and edge $(k, \ell)$ has weight $S_{k,\ell} = \frac{1}{N} \sum_n \frac{\mathbb{1}_{\{k,\ell\} \subset \mathcal{T}_n^c}}{T - K_n}$. One can show that $\sum_{\ell \neq k} -P_{k,\ell} = P_{kk}$
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$$u^\top P u \geq \min_{k,\ell \in (s^*\text{-}t^*)} (-P_{k,\ell}) \frac{(u_{t^*} - u_{s^*})^2}{d_{s^*,t^*}} \geq \frac{4T \|u\|_2^2}{N(T+1)^2(T-1)^2}$$

where $u_{t^*} := \max_t u_t$, $u_{s^*} := \min_t u_t$ and $d_{s^*,t^*}$ distance in $\mathcal{G}$, and $(s^*\text{-}t^*)$ a path from $s^*$ to $t^*$.

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- As $\mathbf{1}$ is an eigenvector of $P$ associated to $\lambda_0 = 0$, from the minmax theorem, we get $\lambda_1(P) \geq \frac{4}{N(T+1)^2(T-1)} := 1 - \rho_{NT}$

# back to the two subproblems...

**Master Problem**

$$\min_{\boldsymbol{p} \in \mathbb{R}^T} f(\boldsymbol{p})$$
$$\text{s.t. } \boldsymbol{p} \in \mathcal{P}^{(s)}$$

**Disaggregation Problem**

$$\text{Find } \boldsymbol{x} \in \mathcal{Y}_{\boldsymbol{p}} \cap \left(\textstyle\prod_n \mathcal{X}_n\right)$$

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**How can we compute $\sum_n x_n$ without disclosing profiles to Big Brother ?**

# Issues: transmission of profiles for projection

In APM, agents still have to provide profiles $(\boldsymbol{x}_n^{(k)})_n$
$\rightarrow$ **Secure Multiparty Computation** (SMC) principle

**Require:** Each agent has a profile $(\boldsymbol{x}_n)_{n \in \mathcal{N}}$
1: **for** each agent $n \in \mathcal{N}$ **do**
2: Draw $\forall t, (s_{n,t,m})_{m=1}^{N-1} \in \mathcal{U}([0,A]^{N-1})$
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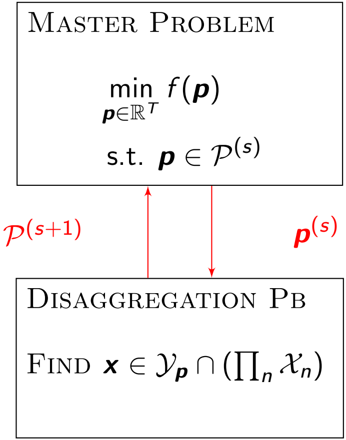
**end**



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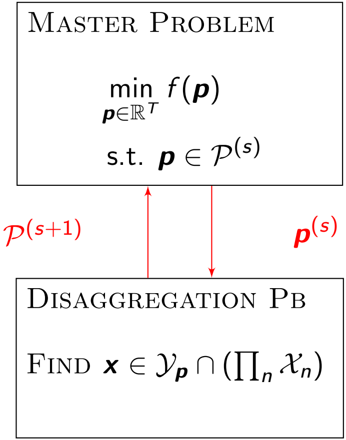
  **end**

  $s \leftarrow s + 1$

**done**



# Termination condition: number of cuts

**Proposition**

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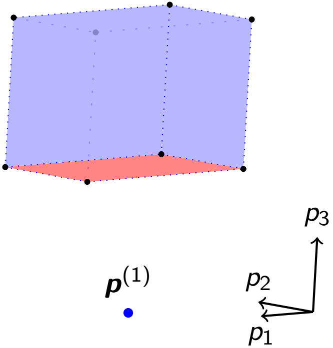
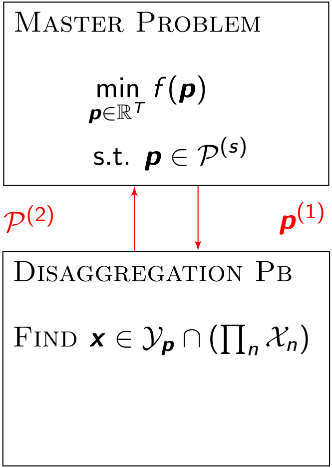
The procedure stops after a finite number of iterations, as at most $2^T$ constraints can be added to the master problem.

ISSUE: we need the limit $\boldsymbol{x}^\infty$ of the APM sequence to obtain the cut..
but in practice we can stop in finite time and obtain the ~~approximated~~ same cut!

# Illustrative example in dimension $T = 4$ (with $\sum_t p_t = \sum_n E_n$)



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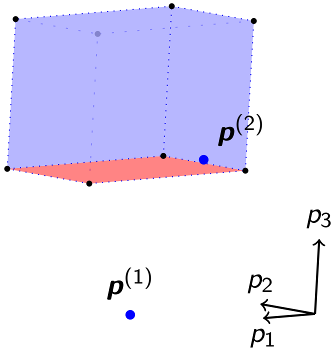


MASTER PROBLEM

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FIND $\boldsymbol{x} \in \mathcal{Y}_{\boldsymbol{p}} \cap (\prod_n \mathcal{X}_n)$

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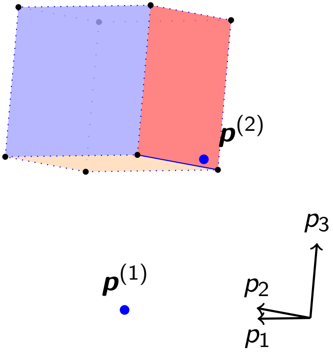


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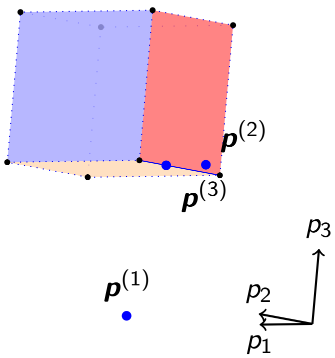


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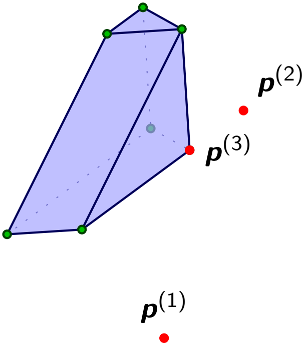


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The method computes a resource allocation $\boldsymbol{p}$ and $N$ individual agents profiles $(\boldsymbol{x}_n)_n$, such that $(\boldsymbol{x}, \boldsymbol{p})$ solves the global (nonconvex) problem, while keeping private:

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**Further work and extensions:**

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**Further work and extensions:**

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- analysis on the maximal number of constraints added (polynomial bound ?).

## THANKS !


Jacquot, Paulin, Olivier Beaude, Pascal Benchimol, Stéphane Gaubert, and Nadia Oudjane (2019a). "A Privacy-preserving Disaggregation Algorithm for Non-intrusive Management of Flexible Energy". In: IEEE 58th Conference on Decision and Control (CDC). IEEE. arXiv: 1903.03053.

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