Demand Response in the Smart Grid: the Impact of Consumers Temporal Preferences

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Introduction: the Context

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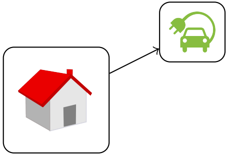

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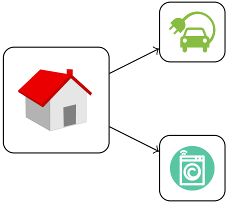

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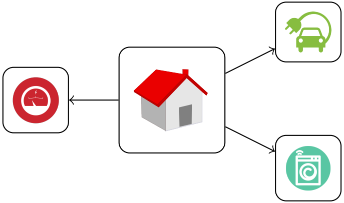

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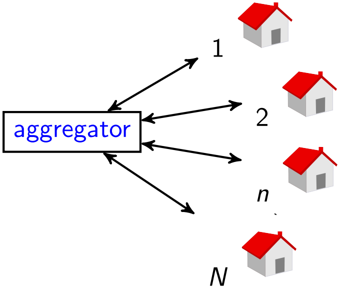

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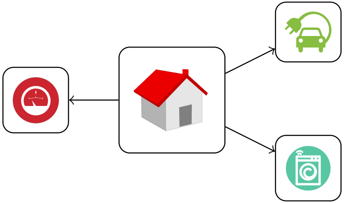

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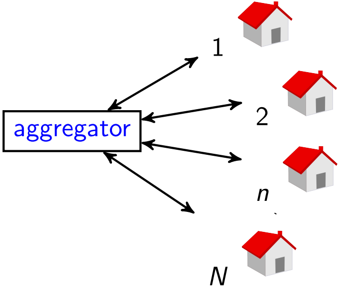

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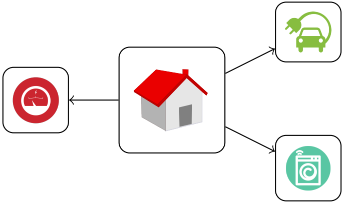

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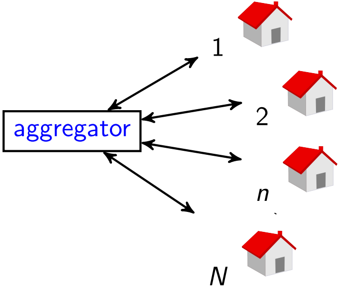

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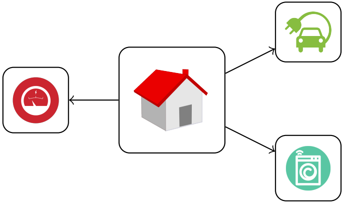

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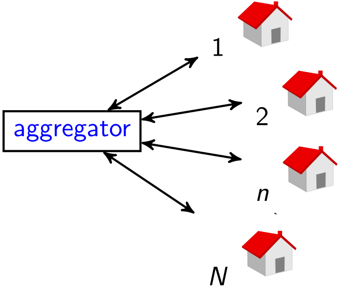

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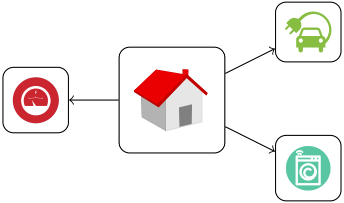

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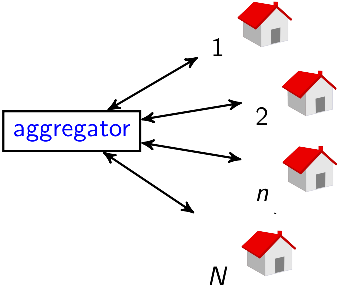

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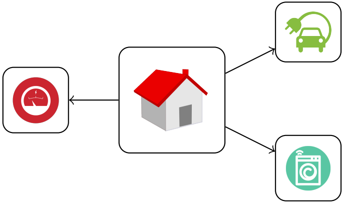

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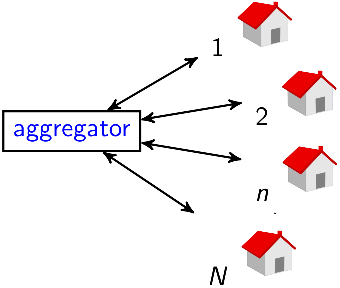

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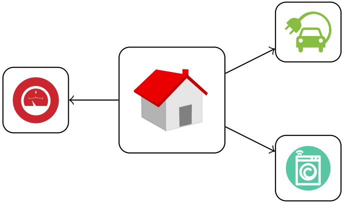

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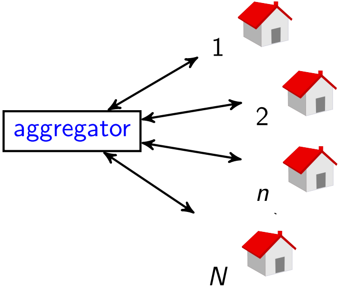

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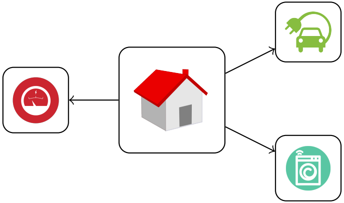

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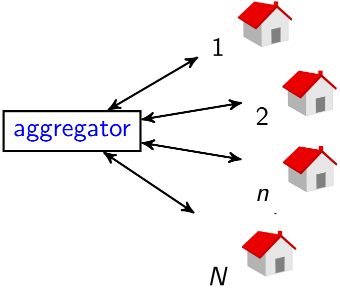

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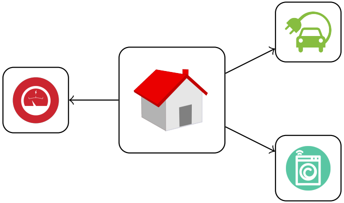

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$\rightarrow$ find a procedure that optimizes system costs AND users costs.

Billing mechanisms and System Costs

Each $n \in \mathcal{N}$ minimizes a *bill* (signal) : $\min_{\boldsymbol{\ell}_n \in \mathcal{L}_n} b_n(\boldsymbol{\ell}_n, \boldsymbol{\ell}_{-n})$ subject to $\begin{cases} \sum_t \ell_n^t = E_n \\ \underline{\ell}_n^t \leq \ell_n^t \leq \overline{\ell}_n^t, \forall t \end{cases}$.

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- **Daily Proportional (DP)** [Mohsenian-Rad et al. (2010)]

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For HP and DP, **system costs** $\mathcal{C}(\boldsymbol{\ell}) = \sum_t C_t(\ell^t)$ are equal to sum of users bills $\sum_n b_n(\boldsymbol{\ell})$.

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How does the parameter α influence the game ?

Nash Equilibrium: Equilibrium Profile

We look for a stable situation where no user wants to change its profile

$\rightarrow$ NASH EQUILIBRIUM (NE)

$$\mathcal{L}_\alpha^{\text{NE}} := \left\{ \boldsymbol{\ell}^{\mathrm{NE}} \in \mathcal{L} : \text{ for each } n, \boldsymbol{\ell}_n^{\mathrm{NE}} \in \arg\min_{\ell_n \in \mathcal{L}_n} f_n^\alpha(\boldsymbol{\ell}_n, \boldsymbol{\ell}_{-n}^{\mathrm{NE}}) \right\} .$$

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With any $\alpha \in [0,1]$ and costs $C_t(\ell) = a_t \ell + b_t \ell^2$, for the games $\mathcal{G}_\alpha^{\text{DP}} = \left(\mathcal{N}, \mathcal{L}, (f_n^\alpha)_n\right)$ and $\mathcal{G}_\alpha^{\text{HP}} = \left(\mathcal{N}, \mathcal{L}, (f_n^\alpha)_n\right)$, the following results hold:

- *each game* **has a unique** *Nash Equilibrum,*
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Remark 2: The BRD provides a **decentralized** algorithm to compute each equilibrium.

Efficiency Metrics

- PRICE OF ANARCHY, (Koutsoupias and Papadimitriou, 1999)
  Define $\mathbb{SC}(\boldsymbol{\ell}) = \sum_n f_n(\boldsymbol{\ell})$ the **Social Cost** and $\mathbb{SC}^*$ the optimal social cost.

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  We define a similar quantity from the system side, without the users preferences, but only considering the **system costs** $\mathcal{C}(\boldsymbol{\ell}) := \sum_{t\in\mathcal{T}} C_t(\ell^t)$:

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Remark: PoA ≥ 1 and PoE ≥ 1. For $\alpha = 0$, $\text{PoE}(\mathcal{G}_\alpha) = \text{PoA}(\mathcal{G}_\alpha)$.

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Theorem

Under certain conditions on $\hat{\ell}_n$ and $(E_n)_n$, the unique NE are given, for $\alpha \in [0,1]$ by:

$$\text{for } \mathcal{G}_\alpha^{DP},\ \ell^P = \frac{E}{2} + \alpha \frac{(\hat{\ell}^P - \hat{\ell}^O)}{2} ,$$

$$\text{for } \mathcal{G}_\alpha^{HP},\ \ell^P = \frac{E}{2} + \phi(\alpha) \frac{(\hat{\ell}^P - \hat{\ell}^O)}{2} .$$

with: $\phi(\alpha) \overset{def}{=} \frac{2\alpha}{(1+\alpha)+(1-\alpha)N} \in [0,1]$.

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$\rightarrow$ Explicit System costs and PoE at NE

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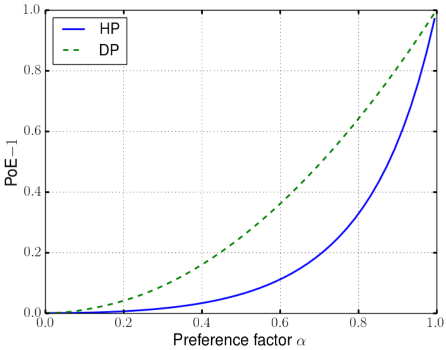

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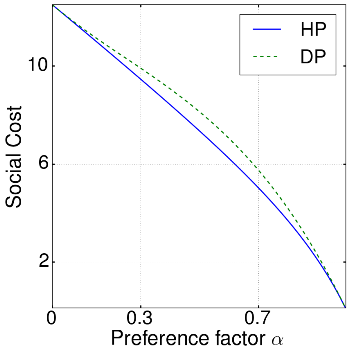

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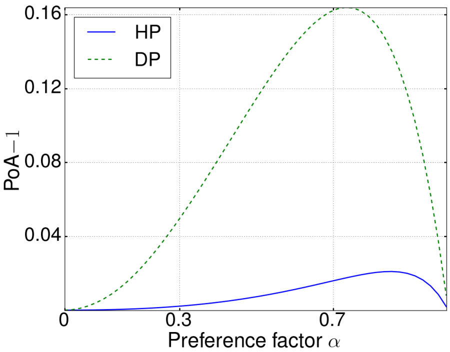
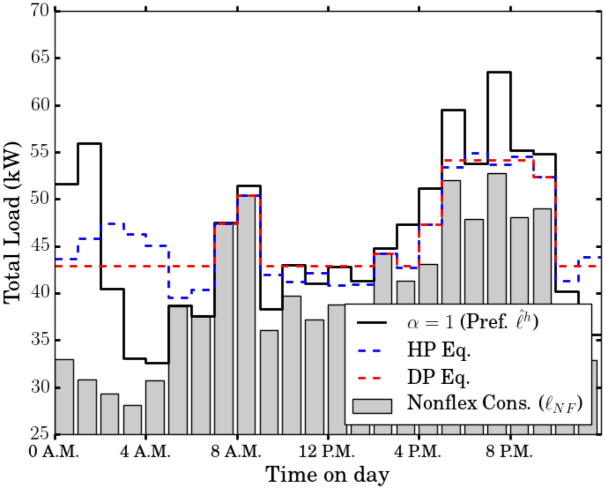


Simulation with 30 users on a real database

Simulation on DR on all days of January 2016.

Example on 2016/01/10:

$\alpha = 0.0$



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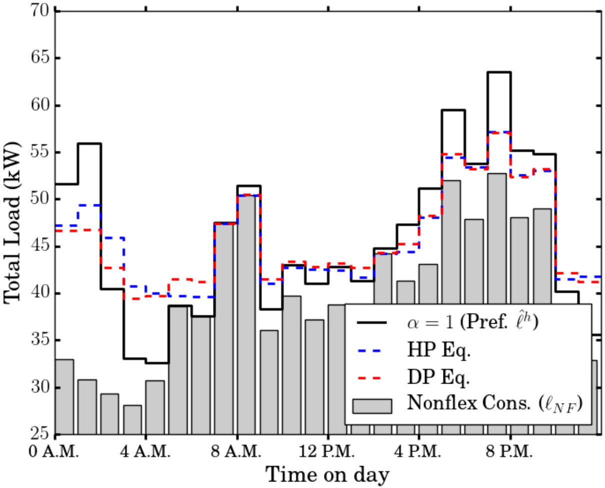

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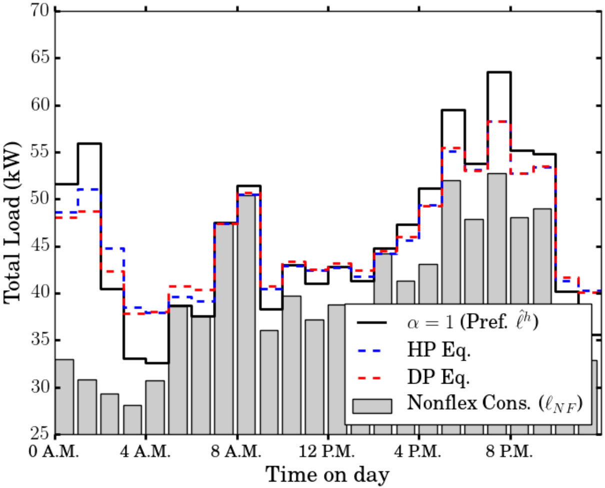

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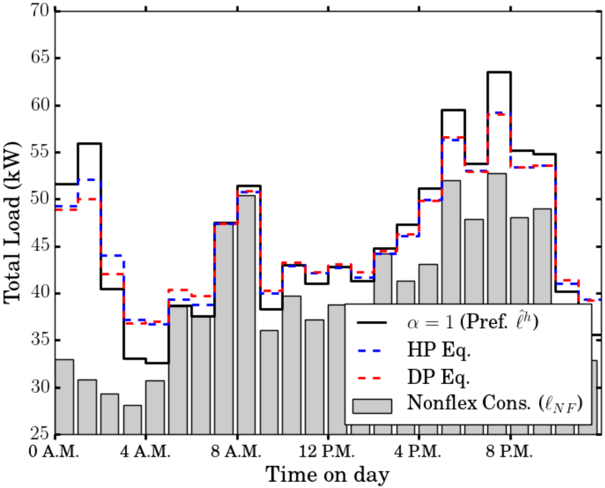

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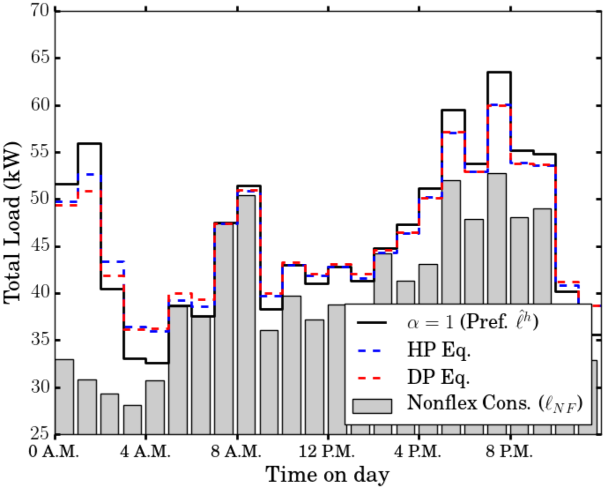

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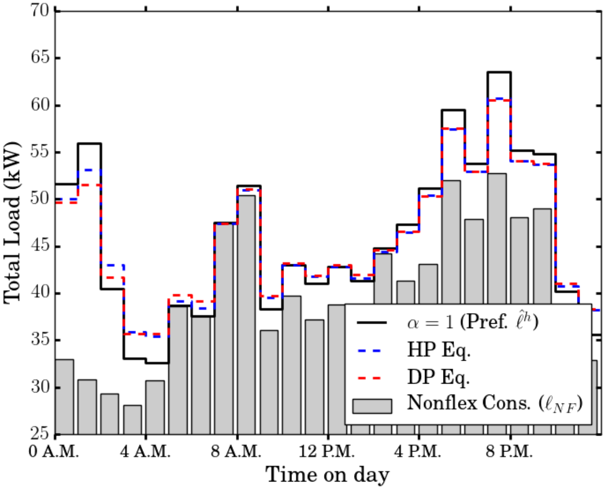

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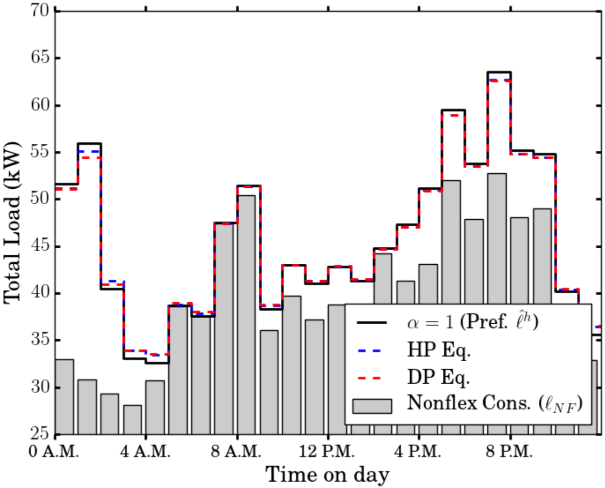

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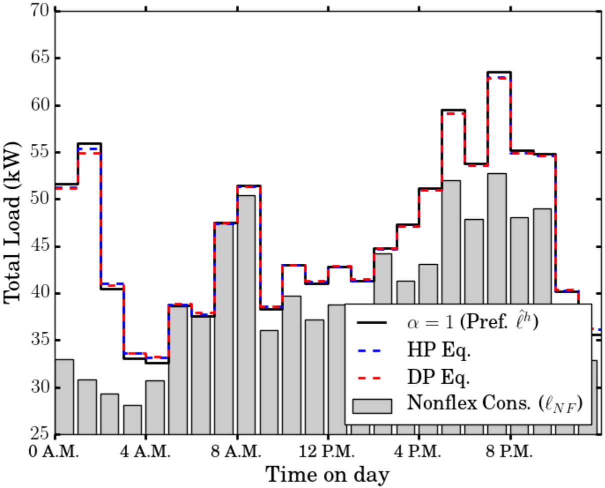

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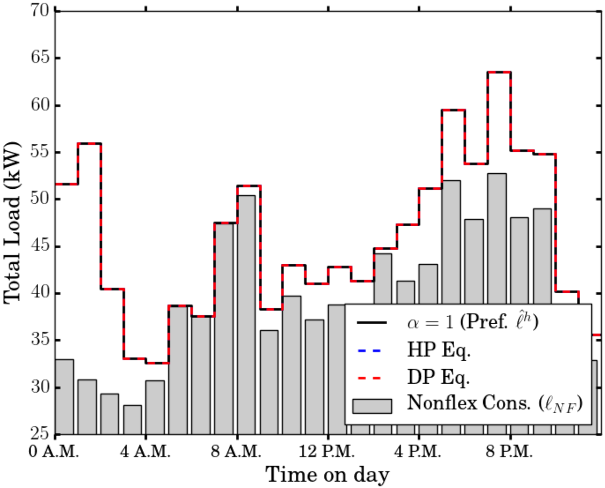
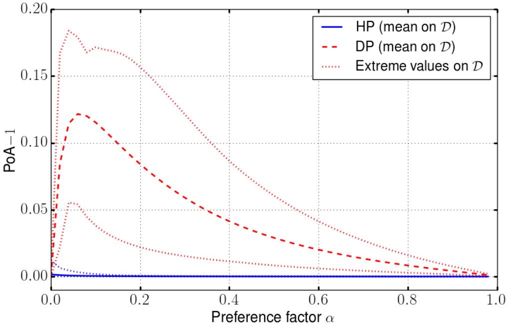


Price of Anarchy



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HP billing numerically better on PoA and PoE

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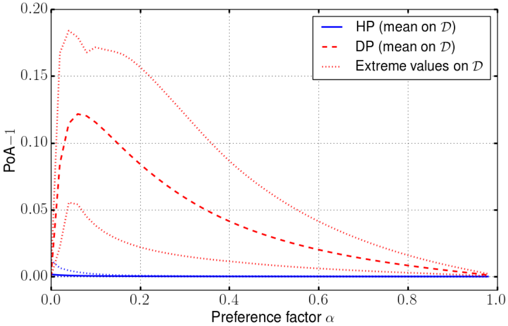

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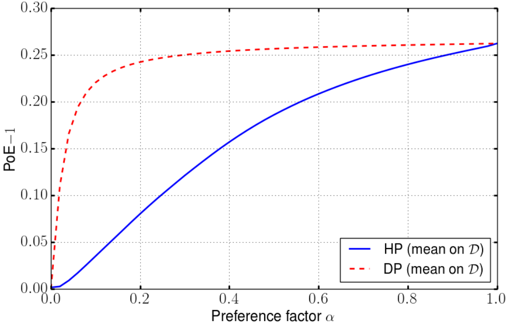

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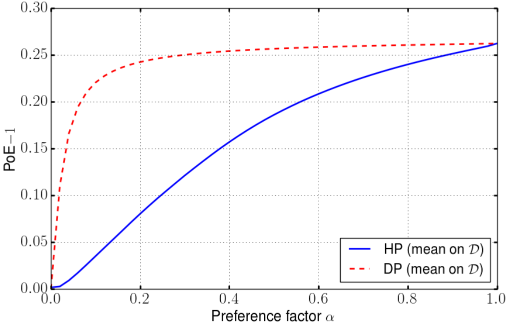

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Conclusion and Perspectives

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THANK YOU!

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References

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