Demand Side Management in the Smart Grid: an Efficiency and Fairness Tradeoff

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ISGT, Torino

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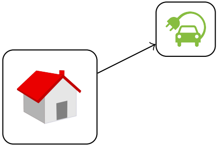

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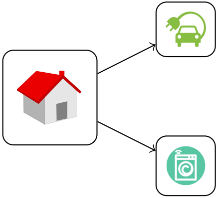

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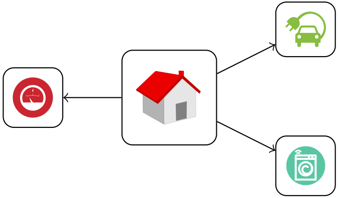

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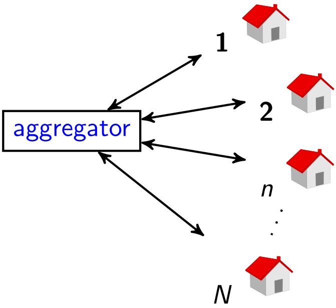

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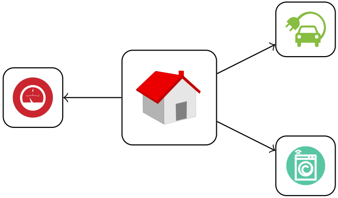

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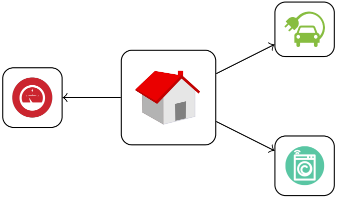

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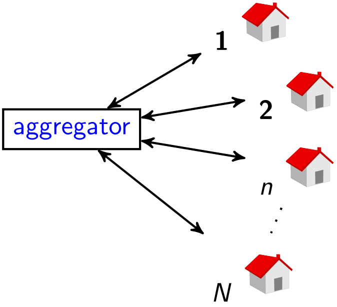

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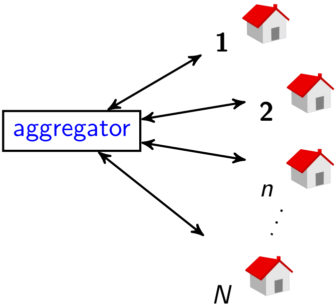

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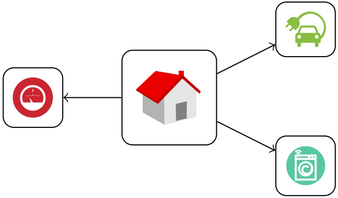

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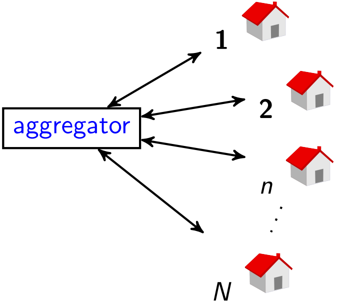

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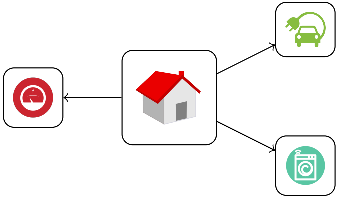

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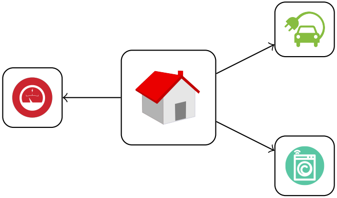

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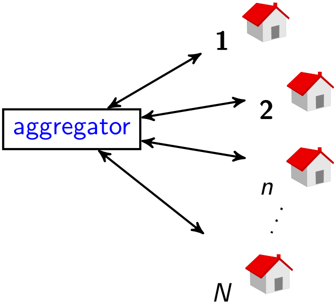

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$\Rightarrow$ Game $\Rightarrow$ Nash Equilibrium .

- **Daily Proportional (DP)** [Mohsenian-Rad et al., 2010]

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  - New Theorem of NE uniqueness,

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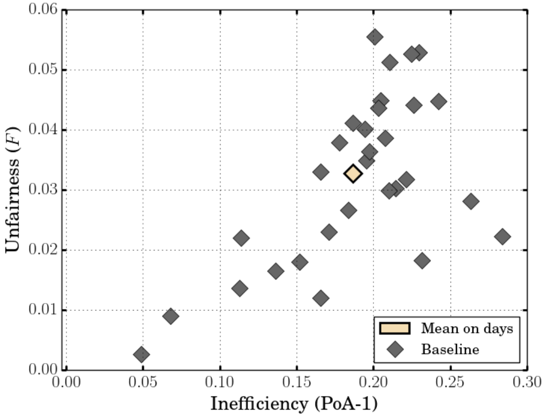
- **Fairness** [Baharlouei et al., 2013]:
  Distance of DSM bills to the vector of “externalities” :

$$F := \text{distance (externalities, bills)}$$

  - Externality(n) = (Optimal cost with n) – (Optimal cost without n) .

Numerical experiments based on real data (*Pecan Street Inc.*) with flexible EV owners (and electrical heating) on 30 days:



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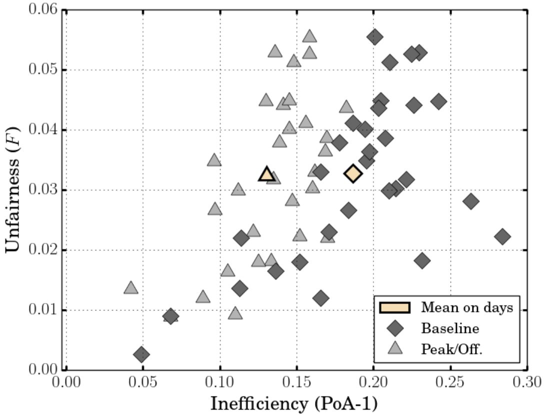

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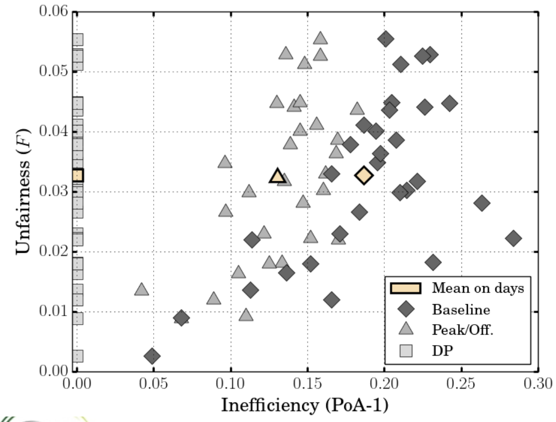

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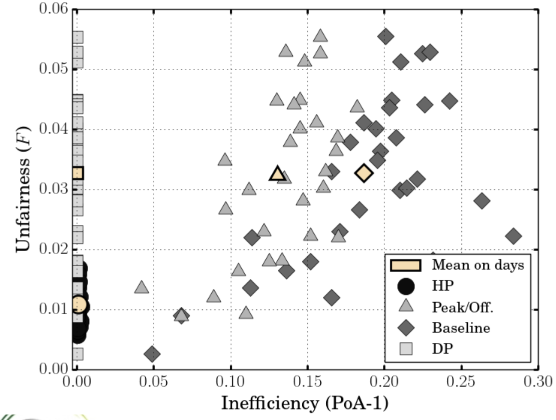

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- HP also very efficient, and has a strong fairness.

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THANK YOU!

[1] Baharlouei, Z., Hashemi, M., Narimani, H., and Mohsenian-Rad, H. (2013). Achieving optimality and fairness in autonomous demand response: Benchmarks and billing mechanisms. *IEEE Transactions on Smart Grid*, 4(2):968–975.

[2] Mohsenian-Rad, A.-H., Wong, V. W., Jatskevich, J., Schober, R., and Leon-Garcia, A. (2010). Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid. *IEEE transactions on Smart Grid*, 1:320–331.

In the complete model, each consumer n can have constraints in his consumption, and solves the optimization problem:

$$\min_{\boldsymbol{\ell}_n} b_n = \kappa \sum_t \left(\frac{C_t(\boldsymbol{\ell}^t)}{\sum_m \ell_m^t} \cdot \ell_n^t \right) \tag{1a}$$

$$\sum_{t \in \sqcup} \ell_n^t = E_n, \tag{1b}$$

$$\underline{\ell}_n^t \leq \ell_n^t \leq \overline{\ell}_n^t, \ \forall t \in \mathcal{T} \ . \tag{1c}$$

For any subset of consumers $\mathcal{M}$, consider the induced optimal total cost:

$$\mathcal{C}^*_{\mathcal{M}} := \inf_{(\ell_m)_{m\in\mathcal{M}}} \sum_{h\in\mathcal{H}} C_h \left(\sum_{m\in\mathcal{M}} \ell^h_m\right) ,$$

and define the externality of player n as the cost induced on the system by n:

$$V_n := \mathcal{C}^*_{\mathcal{N}} - \mathcal{C}^*_{\mathcal{N}\setminus\{n\}}$$

Then, we define the fairness of the DSM billing mechanism as:

$$F := \sup_{\mathbf{x}\in\mathcal{X}^{\text{NE}}_{\mathcal{G}}} \left[\sum_{n\in\mathcal{N}} \left| \frac{V_n}{\sum_{m\in\mathcal{N}} V_m} - \frac{b_n(\mathbf{x})}{\sum_{m\in\mathcal{N}} b_m(\mathbf{x})} \right|\right] .$$

Theorem (Uniqueness of a Nash Equilibrium with HP billing)

Let $c_h(\ell^h) := \frac{1}{\ell^h} C_h(\ell^h)$ *be the per-unit price of electricity. If* $c_h' \geq 0$*, i.e. prices are increasing with global load, then a Nash Equilibirum exists. If, in addition:*

$$\forall h, \frac{(\ell^h)^2}{\sum_n (\ell_n^h)^2} > \left(\frac{\ell^h c_h''(\ell^h)}{2 c_h'(\ell^h)} \right)^2$$

then the Nash Equilibrium is unique.

Theorem (Bound on the Price of Anarchy with HP billing)

In the case of costs of the form $C_h(\ell^h) = a_2^h (\ell^h)^2 + a_1^h \ell^h$ *, the price of anarchy is bounded by:*

$$PoA \leq 1 + \frac{3}{4} \sup_{h \in \mathcal{H}} \frac{1}{1 + a_1^h / (a_2^h \bar{\ell}^h)} .$$